# The Appreciation and Volatility of Price in Different Classes of Housing

*by Michael G. Ferri and L. Randolph McGee*

This paper explores the relationships between the characteristics of housing (age, size, and so on) and changes in the price units over time. Important findings can be summarized briefly at the outset. The price of what must be called the "better unit"—the bigger, or newer, or more luxurious house—is subject to greater change in price than the smaller and less well-appointed house. Furthermore, the price of the better unit is more sensitive to general housing conditions, indicating that the less luxurious unit has greater price stability across the upturns and downturns of the economy.

The analysis was motivated by an unusual gap in the research on housing economics. This research provides abundant information about the relationship between a house's attributes and its price, but there has been little investigation on the association between price changes and characteristics. For example, it is easy to find how much an additional bath may add to the price of a house, but difficult to locate any information about whether houses with more baths might increase faster or change prices more quickly than units with fewer baths.

The research was carried out with tools borrowed from modern portfolio theory and investment analysis. This field has been preoccupied with variation in the prices and returns of financial securities and has devised a set of price change measures that can be very instructive of the cause and degree of price volatility. In fact, some of these measures have already been applied to certain problems in real estate

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economics. This paper complements those earlier efforts and demonstrates the increasing usefulness of portfolio-oriented techniques to the study of housing price movements.

## A BRIEF REVIEW OF PORTFOLIO THEORY

The eventual outcome of an investment is never known with certainty. An investor can know, at best, only the list of possible rates of return on his investment and the relative likelihood (or probability) of each rate. The pairing of outcome and probability is a probability distribution, a tool which facilitates investment analysis. The expected return from the investment is given by the mean of the distribution. The project risk is represented by the dispersion of possible outcomes around the mean. Investments with widely flung possible rates of return are risky, in the sense that the investor cannot be very confident about his eventual profits. Projects with possible returns clustered near the mean are relatively safe investments because the investor has only minor uncertainty about the eventual rate.

Though several dispersion measures are available, modern portfolio theory has adopted the standard deviation of returns as its chief measure of dispersion and, hence, investment risk. This theory, associated with the works of Markowitz and Sharpe, views the standard deviation of returns as the total risk of a project. But risk has been refined by partitioning that total risk into two components. The first is unsystematic risk, caused by factors unique to the asset. The unsystematic risk of an equity share, for example, is most usually attributed to the firm's management, capital structure, or to its regulatory environment. Systematic risk is that variation in return which may be caused by factors prevading the entire economy and affecting the returns of all assets. Interest rate movements, inflation, and political events are commonly cited causes of systematic risks in equities.

The separation of total risk into systematic and unsystematic risk is accomplished by means of regression analysis. Regression is a statistical tool for determining how much one variable (the dependent variable) will change in response to a fluctuation in a second variable (the independent variable). When change in the dependent variable is a result of one independent variable, regression amounts to drawing a line through points which are pairs of values of the two variables. Such a line cannot intersect every point, because of random and unpredictable disturbances in the economy. But the line can be drawn so that it is as close as possible to all of the points. The line that minimizes the differences between itself and the various points is the "best" fit and gives an algebraic representation to the relationship between the two variables.

Portfolio analysis utilizes this technique by employing an average market return or the return on some market index as the independent variable and relating it to the return of an individual security. The return from a market index will adequately capture the influence of factors that pervade the economy and cause the prices (and returns) of all assets to vary—the definition of systematic risk. So, the relationship between this market and some security indicates the systematic risk of that security. Any remaining risk is unsystematic, due to factors peculiar to the firm and not to factors that may affect the performance of all firms.

The application of regression analysis to portfolio theory can be further explained by use of the following *Equation (1)*:

$$(1)\quad R_{it} = \alpha_i + \beta_i R_{mt} + e_t$$

Here, $R_i$ is the return (gathered for a number of periods) of asset i, $R_m$ is the average return on a large number of assets; e is the random shock or disturbance in a time period, and the number of t's or time periods from which the data can be gathered is largely a function of the scope of the inquiry at hand. In analysis of equities, the return on the asset is defined as capital appreciation in the period and any dividend payment in that period divided by the price of the preceding period. The rate of return on the market is defined as the change in the average price of the assets plus the average dividend, divided by the average price of the preceding period. *Equation (2)* gives a formal definition of these rates:

$$(2)\quad (a)\quad R_{it} = (P_t - P_{t-1} + D_t) / P_{t-1}; \text{ and}$$

$$(b)\quad R_{mt} = (V_t - V_{t-1} + D^*_t) / V_{t-1}$$

where P is the price of the asset, D is its dividend, V is the average value of the group of stocks that make up some index, and D* is the average dividend of the stocks in the index. Standard and Poor's 500 Common Stock Index is commonly used to represent the price of the "market" for equities and the measure of the impact of systematic factors in the economy.

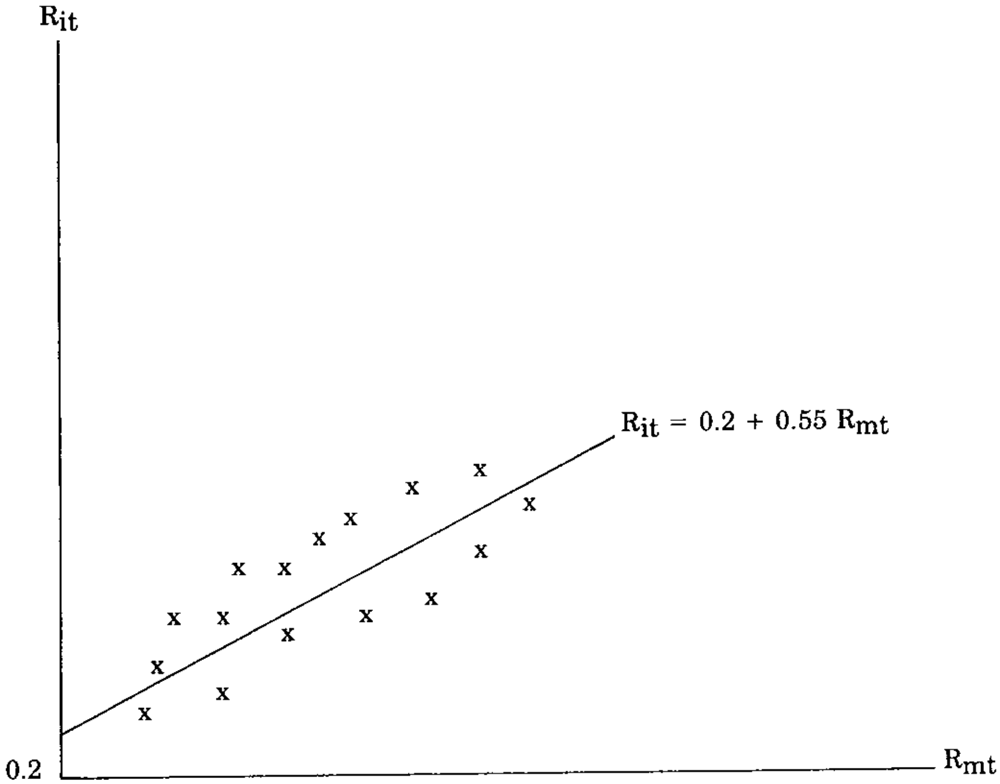
It should be noted that regression analysis will give estimates of the values of $\alpha$ and $\beta$ in *Equation (1)*. The regression intercept, $\alpha$, is the average return on asset i when the market is unchanging (or when $R_{mt}$ = 0). The slope coefficient is $\beta$ (commonly called "beta").[1] Beta measures the fluctuation in the asset's return for a given unit of change in the market's average return. *Value Line's* volatility index for stocks is one practical application of the beta concept. Beta may be formally defined as the covariance between the asset's return and the market's return, divided by the variance of the market's return:

$$\beta_i = \mathrm{Cov}(R_i, R_m) / \sigma^2_{R_m}$$

*Equation (1)* is called the capital market line or the market model and is graphically represented by *Exhibit 1.*

**EXHIBIT 1**

THE MARKET MODEL FOR A HYPOTHETICAL SECURITY




The x's are pairs of $R_i$ and $R_m$. $\beta$ is 0.55; $\alpha$ is 0.2.

Beta is an index of the sensitivity of an asset's return to movement in the returns of the entire set of assets to which it is similar and with which it forms a "market." Thus, beta coefficients make possible an ordinal ranking of assets on their systematic risk. Assets with high betas (greater than 1.0) are very sensitive to and actually more volatile than the average asset or the market itself. For example, an asset with a beta of 1.3 will have a change in returns that exceeds that of the market by 30 percent when the market's general value is rising and that fall 30 percent more than the market's overall average when the market is in decline. Assets with betas in the neighborhood of 1.0 have patterns of return that are similar to the market's general movements.

An asset with a beta of less than 1.0 will be less volatile than the market. That asset will rise less than the market does and fall less than the market does. This asset's systematic risk is said to be relatively low. Betas may be estimated for any interval—week, month, quarter, and so on—for which the necessary data is available. The required data consists simply of returns on the asset and returns on a representative market average.

Portfolio theory supplies two measures of risk—beta and standard deviation—for the analysis of risk and return on housing investments. Standard deviation represents the total variability in the return, and beta gives an ordinal ranking of the assets on their responsiveness to changes in general market conditions. In a later section of this paper, these measures of risk, and a measure of rate of return, will be estimated for different types of housing.

## THE MARKET MODEL AND HOUSING

According to some analysts, certain features of housing and real estate argue against the use of the market model. The multitude of localized real estate markets is said to make identifying the "relevant" market, for the application of portfolio analysis, a difficult task. The possibility that local markets are segmented according to the income of buyers and/or the location of individual properties may make identifying the proper market even more difficult. Another troublesome feature of the real estate market is the infrequency of sale of any one unit and the resulting discontinuous price history of particular houses. An additional pitfall may be the long planning horizon of participants in the market who view their purchases as capital assets and not as liquid investments. Finally, the use of the market model requires a great deal of data; real estate data from any specific local market might be insufficient.

The search for the relevant market should be resolved by the following consideration: the appropriate market for evaluating an asset's systematic risk is the largest market in which it and similar assets are traded. Market model studies of equities employ an index of the average price for the nationally unified market for stock, and similar studies of bonds use indexes of the price level in the national market for bonds. The national stock and bond markets are considered relevant in these studies precisely because the price of a stock or bond is unaffected (save for local taxes, perhaps, or minor frictions of transactions costs) by the location of the asset or the participants. Indeed, there is a unified market for bonds and for stocks. But houses are not liquid financial assets. The price of a house is obviously and directly influenced by its locale or region. The appropriate market index would be that index that reflects the average price level of housing in the relevant region or locale. It should be noted that hedonic studies[2] of the value of individual attributes of housing are confined (generally) to local markets: cities, counties, or metropolitan areas. It seems acceptable that studies of the relative price fluctuations and increases of houses with different characteristics should have a similar focus.

Possible segmentation of the local markets along lines of the income of buyers or the location of the units should also be considered in light of the highly successful hedonic studies. Restricted to particular local markets, these studies clearly assume the existence of local arbitragers who drive the marginal prices of attributes to some appropriate level. Another way of stating this crucial assumption is to posit such a variation and richness of incomes and preferences (and other features of assets and participants) that the real estate market becomes continuous for the important characteristics of housing. In either case, the hedonic approach relies on the assumption that the lines of segmentation are not pronounced, or at least not enough to fragment the local market. Thus, it seems proper to conclude that an analysis of the price volatility of housing of different kinds (and attributes) should be based on the same conclusion: that the local market is sufficiently unified to provide unbiased estimates of the price of attributes and of the relative price changes of housing with different attributes.

The frequency with which individual housing units are traded is another concern. Unlike many financial assets, most houses are traded infrequently. The market model cannot be applied to the price history of specific housing units with any ease because that history is not likely to be rich enough in recorded prices. The model can be extended to broadly defined classes of housing, in terms of age or size of a unit for example. A continuous price history can be created for such classes, which could be based on the average price of the particular units in each class that do change hands in a period and across a number of periods. In other words, one can develop the measures of total and systematic risk only for certain types of housing, but not for individual units. One may be able to show, for example, that houses between six and ten years old have greater total and systematic risk than younger houses, but unfortunately it would be close to impossible to estimate the risks of some given seven-year-old unit as opposed to the market risks of a certain three-year-old house.

Extending the market model to housing requires an index of the price of all housing, representing the value of the "market" for housing. On this basis it is possible to define the return, in each period, to housing in general and to specific classifications of housing. It is also possible to define the average return, across a number of periods, for housing and each type of unit. Finally, it is possible to estimate both the total and the systematic deviations in returns (through the use of the market model) for each type of housing.

Long planning horizon of many participants in the real estate market is another problem. However, numerous markets share this characteristic also, and many buyers of bonds, stocks, and other assets anticipate a long holding period. But each of these markets eventually becomes continuous, both in terms of supply and demand, precisely and especially because few participants have the *same* plans. The multitude of preferences regarding time creates trading in which one can find few sharp cleavages and in which continuity is the rule. Another feature of all markets is the existence of arbitragers who can alter any anticipated holding period for obvious gains from special transactions. There are many brokers, Realtors, and financial institutions who fit this description and who would supply needed continuity in demand and supply of housing in the rare event that the greatly varied horizons of the multitude of participants do not provide a continuous market.

## DATA AND INDEXES OF HOUSING PRICES

The data for this project consist of 2,746 observations on the sale of single-family, detached, residential units in Lexington (Fayette County), Kentucky, over the interval from January 1971 to December 1975. The data have been previously employed in a study of hedonic prices and hedonic indexes.[3] Each observation in this set contains information on several important characteristics of the basic housing unit. Of particular importance in the following tests are: 1) age in years, 2) the presence or absence of central air conditioning unit, 3) residential space in square footage, 4) the size of the lot in square footage, 5) the presence or absence of a garage, 6) the number of baths, in terms of half-baths (1.0, 1.5, etc.), 7) sale price in dollars, and 8) the time of sale in months (and quarters) and years. The data were gathered from the Multiple Listing Service of the Board of Realtors of Lexington, and represent a very large sample of the recorded sales in this period of time.

From this data, 24 quarterly-price indexes were created which cover the period from 1971-I to 1975-IV. The value of any index in a quarter is the mean sale price of the group of houses of a particular type sold in the quarter. One index is the average price of all units sold; this grand mean functions as the index of the value of the market for housing. Five indexes were created on the basis of the age of the house. There are price indexes for housing that is less than or equal to five years old, housing aged between six and ten years, and so on. Two price indexes were created on the basis of the presence of a central air conditioning unit; one index plots the average price of houses that have such a unit and the other index pertains to the price of houses without this amenity. Two indexes pertain to the presence of a garage. Three pairs of average prices were created on the basis of the size of a house or its residential space. In each pair, houses were assigned to one or the other of the indexes on the basis of whether they have more or less than 1200 square feet, 1350 square feet, and 1500 square feet, respectively. Two pairs of averages were based on the number of baths: for the first pair, the dividing line was 1.5 baths; for the second, the distinction was whether the house had more than two full baths. Finally, two pairs of indexes were created according to whether the size of the lot exceeds 10,000 square feet and 12,000 square feet. It was not possible to create additional, clearly-defined classes of housing because of insufficient data. As it is, the samples on which these various indexes were built overlap one another since one unit's price might be included in five or ten or more of the indexes. This poses no problem, however, because comparisons of risk and return will be made only within the mutually exclusive categories determined by the individual characteristics. The aim is to find the amount of risk inherent in individual characteristics.

Each index is used to create a quarterly rate of return. The formula for this rate of return is given by

$$(3)\ R_t = (P_t - P_{t-1}) / P_{t-1}$$

where P is the average price, or indexed value, and the subscript denotes the present or the previous period. This index ignores any periodic payments, whether they are outflows in the form of maintenance or taxes, or inflows in the form of tax savings. Such payments may differ, of course, according to the age, appointments, or size of the house, but are difficult to measure in any consistent manner. Further, several studies of the market model reveal that price changes (capital gains) are the driving force in the rate of return and represent the bulk of systematic variations in returns.

A rate of return was computed for the entire market, labeled $H_m$. A similar rate was constructed for each of the other indexes which deal with the houses of a particular type. The rates will be labeled $H_i$—there are 23 of them. For the five years (1971 to 1975), 19 quarterly rates of return were compiled for each index. These observations were used to estimate the linear relationships of the market model:

$$(4)\ H_{it} = \alpha + \beta_i H_{mt}, \qquad t = 1971\text{-I to } 1975\text{-IV}$$

These observed rates of return were also used to compute the standard deviation and the mean quarterly return for each index. These statistics are reported in *Exhibit 2,* along with the size of the sample in each category and, the $R^2$ for the regression. The size of samples in the types of housing preclude domination by any one category of the entire market. Further, no alpha was reported because no estimate was significantly different from zero.

## RISK AND RETURN FROM TYPES OF HOUSING

The statistics reported in *Exhibit 2* should be interpreted in the following manner. Houses that are less than five years old, for

## EXHIBIT 2

### RISK AND RETURN ON TYPES OF HOUSING: 1971-II to 1975-IV; LEXINGTON, KENTUCKY (1)

| | | Risk | | | Regression Statistics (3) |
|---|---|---|---|---|---|
| Type of House | Sample Size | Systematic (2) (Estimated Beta) | Total (%) Standard Deviation of Returns | Mean Return (%) | Explanatory Power — $R^2$ |
| **AGE, IN YEARS** | | | | | |
| 5 or less | 819 | .75 (.22)* | 8.9 | 2.1 | .414 |
| 6-10 | 716 | 1.33 (.24)* | 12.6 | 3.2 | .653 |
| 11-15 | 500 | 1.45 (.25)* | 13.7 | 2.9 | .660 |
| 16-20 | 322 | -.03 (.45) | 14.2 | 4.2 | .000 |
| 21 and more | 389 | -.04 (.45) | 14.3 | 3.1 | .000 |
| **CENTRAL AIR** | | | | | |
| Yes | 1,415 | 1.03 (.26)* | 11.3 | 1.7 | .485 |
| No | 831 | .73 (.17)* | 7.7 | 1.8 | .522 |
| **RESIDENTIAL SPACE, IN SQUARE FEET** | | | | | |
| < 1200 | 1,237 | .34 (.27) | 8.7 | 1.9 | .088 |
| ⩾ 1200 | 1,509 | .82 (.11)* | 7.2 | 2.3 | .751 |
| < 1350 | 1,497 | .39 (.24) | 8.1 | 2.1 | .135 |
| ⩾ 1350 | 1,249 | 1.01 (.14)* | 8.9 | 2.5 | .767 |
| < 1500 | 1,814 | .33 (.23) | 7.6 | 2.1 | .110 |
| ⩾ 1500 | 932 | 1.00 (.16)* | 9.1 | 2.5 | .711 |
| **LOT SIZE, IN SQUARE FEET** | | | | | |
| > 10,000 | 1,413 | .84 (.21)* | 9.1 | 2.8 | .491 |
| ⩽ 10,000 | 1,333 | .49 (.13)* | 5.6 | 2.6 | .448 |
| > 12,000 | 1,895 | .92 (.09)* | 7.6 | 2.5 | .858 |
| ⩽ 12,000 | 851 | .43 (.22)* | 7.7 | 2.6 | .176 |
| **GARAGE** | | | | | |
| Yes | 1,665 | 1.37 (.10)* | 10.9 | 3.1 | .918 |
| No | 1,081 | .13 (.34) | 10.7 | 2.3 | .008 |
| **BATHS** | | | | | |
| > 1.5 | 932 | .79 (.22)* | 9.4 | 2.2 | .412 |
| ⩽ 1.5 | 1,814 | .25 (.17) | 5.8 | 2.4 | .109 |
| > 2.0 | 335 | .96 (.25)* | 10.8 | 2.7 | .462 |
| ⩽ 2.0 | 2,411 | .43 (.09)* | 4.3 | 2.4 | .882 |
| **AVERAGE FOR ENTIRE MARKET** | 2,746 | 1.00 | 7.6 | 2.4 | — |

(1) No alpha estimate was statistically significant.
(2) Numbers in parentheses give standard error of estimate;
 * denotes significant statistical relationship at 5% level of confidence.
(3) Additional information about the statistical results (such as the Durbin-Watson statistics) can be obtained directly from the authors.

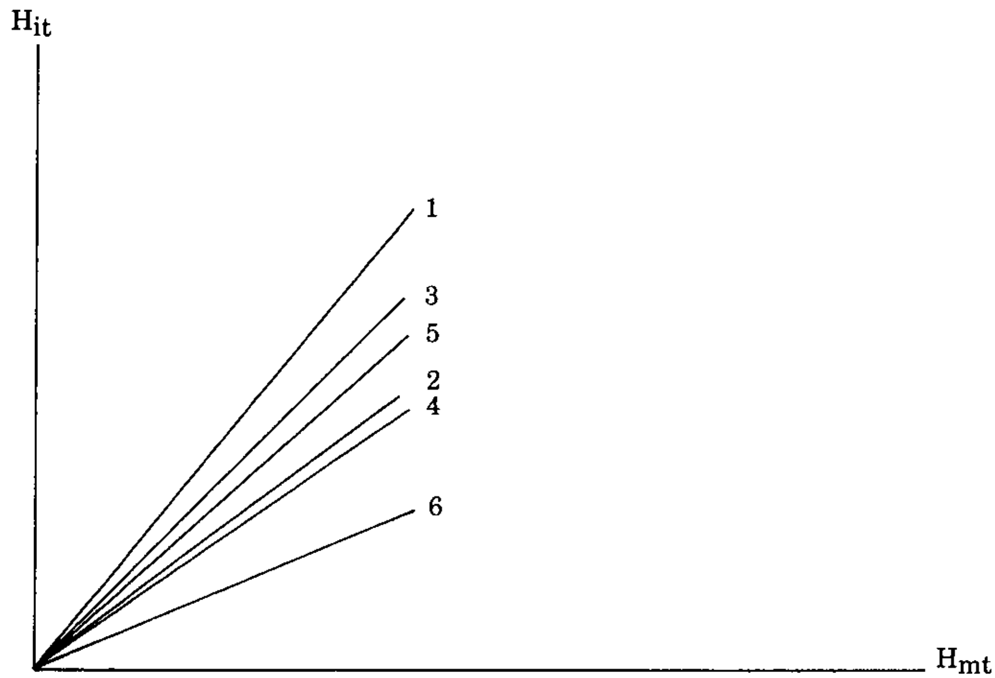
example, have a beta of .75, indicating that a 1.0 percent rise in the average price of housing leads to an increase of 75 basis points in the average price of relatively new units. Houses with garages have a mean quarterly return of 3.1 percent. Houses with central air units have a total risk (standard deviation) of 11.3 percent per quarter.

The results of *Exhibit 2* appear to suggest that houses with what should be considered attractive features have greater systematic risk than houses without those characteristics. For example, younger houses (those aged less than 15 years) have greater betas than the older units. The younger units have betas that range between .75 and 1.45; the older houses have betas quite close to zero. Houses equipped with central air conditioning units are riskier, in the systematic sense, than houses without those facilities. The beta of the first group is 1.03, and the beta of the second is only .73. The association between higher systematic risk and attractiveness of the unit asserts itself under each definition of size of the house and with regard to the size of the lot. And it should be noted that houses with more baths have larger betas (.79 for houses with more than one and one-half baths and .96 for units with more than two baths) than houses with fewer such facilities (.25 and .43, respectively). Houses with garages have greater systematic return risk (beta is 1.37) than houses without garages (beta of .13). The one ambiguity arises in the case of age, where very young houses (those less than five years old) are less risky (their beta is .75) than units whose age ranges from six to sixteen years (beta of these units is at least 1.33). A possible explanation for the higher betas of the moderately aged units is that there is an optimal age for houses, an age where seasoning, maturity, and vintage (the time and technique of construction) compensate for the greater depreciation of the older unit. If this explanation is plausible, then it can be concluded that the systematic risk of housing appears to grow with the addition of desirable characteristics and attributes: larger size, more facilities, and so on. The price movements of the better houses, in short, are more responsive to changes in the returns on the entire market, and the price changes of the less attractive houses are insulated to some extent from market conditions. For selected types of housing, these relationships are graphed in *Exhibit 3.*

Total risk, represented by the standard deviation of returns, is also presented in *Exhibit 2.* In some cases—most notably those associated with air conditioning, the presence of a garage, the number of baths, and size of lot—total risk follows the pattern already observed in the relationships of systematic risk, that is, total risk is greater among the more attractive units. For example, the standard deviation of returns in housing with central air is 11.3 percent, while the total risk of unit without central air is 7.7 percent. In two of the three cases associated with residential space (according to whether or not the unit has more than 1350 or more than 1500 square feet), the same pattern is evident. However, in the case of age and of the remaining comparison by residential space, total risk is larger for what would be the less-

**EXHIBIT 3**

CAPITAL MARKET LINES FOR SEVERAL TYPES OF HOUSING: 1971-II to 1975-IV




1. Houses aged between 11 and 15 years: $H_i = 1.45\ H_{mt}$
2. Houses aged less than 6 years: $H_i = .75\ H_{mt}$
3. Houses with air conditioning: $H_i = 1.03\ H_{mt}$
4. Houses without central air: $H_i = .73\ H_{mt}$
5. Houses larger than 1200 square feet: $H_i = .92\ H_{mt}$
6. Houses of less than 1200 square feet: $H_i = .43\ H_{mt}$

Note: No estimate produced a significant intercept ($\alpha$).

attractive unit. The standard deviation of returns for housing that is smaller than 1200 square feet is 8.7 percent, while the standard deviation in returns for larger housing is 7.2 percent. Also, the standard deviation in returns rises with the age of a house and reaches its highest point with the oldest units, those whose age exceeds 20 years. Though the evidence is not entirely consistent, it does seem to point to a positive association between total return variation and the attributes of size and luxury that make a house attractive.

*Exhibit 2* also contains data on the mean returns to classes of housing. Portfolio theory suggests that mean returns should be positively related to systematic risk, or possibly to total risk. But the pattern of returns does not seem to follow that of systematic variation or of total variation. Houses with central air units, for example, have lower mean returns (1.7 percent per quarter) than houses without such facilities (whose mean return is 1.8 percent), even though the latter type has lower systematic and less total risk than the former. By contrast larger houses, which are riskier in the systematic sense, have greater returns (2.4 percent on average) than do the smaller units which have mean returns near 2.1 percent. Wooden houses, which are less risky in both senses, post lower returns than other types of housing. In general, though all returns to all classes of housing are high these statistics do not present evidence to suggest what causes the difference in returns among the various kinds of units.

## CONCLUSIONS AND IMPLICATIONS

Some tools of modern portfolio analysis have been applied to the price appreciation and the price volatility of housing. According to the study, the house which must be called the more attractive (i.e., the unit with greater size, more facilities, and so on) has more potential for price increases and for price fluctuations than the less well-appointed and smaller house. The greater price variability is of the market-related or systematic type. The result is readily explainable in terms of the shelter value of the units. The bigger and better equipped units provide shelter, as do the more modest houses; but the better units also provide an increment of luxury that the less well-appointed houses do not supply. The value of this additional luxury is more dependent upon market conditions—the level of income and the basic rate of interest—than is the value of shelter alone. Therefore, the more luxury a unit offers, the more sensitive it is to market forces. The percentage price changes in the less well-equipped units are smaller, relative to the market's movements, because the value of shelter (which constitutes a greater part of their overall value) is less susceptible to these market fluctuations.

The results strongly suggest that the better housing units are likely to increase in price during an upswing in the market faster than the average of all houses and than the units of lesser quality. During a period of decline in housing prices, however, the lower quality houses can be expected to experience relatively less decrease in price than either housing in general or the better kind of house. Thus, the less well equipped and smaller units offer some safety of principal.

More than just the marginal value of selected attributes of housing at some point in time can be learned. The potential price increase (or decrease) and the possible market-related price fluctuations of different classes of housing, where classes are based on the amount of certain quantifiable attributes (such as age, size, etc.) or on the presence and absence of certain qualitative characteristics (such as central air units, garages, and so on), can be tracked. The tools and analytical reach of real estate economics (and finance) can be enhanced, it appears, by recognition of this new and until now neglected capability.

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1. Risk may be partitioned in the following way:

   a) $Var(R_i)$ = the total risk of the $i^{th}$ asset;

   b) $= Var(\alpha_i + \beta_i R_m + e)$;

   c) $= Var(\beta_i R_m) + Var(e)$, since the $Var(\alpha) = 0$;

   d) $= \beta_i^2 \times Var(R_m) + Var(e)$, since $\beta_i$ is a constant; and

   e) = systematic risk + unsystematic risk

2. A complete description of the data and its sources, as well as of the price effect of each characteristic, is available in Ferri.

3. Hedonic studies are efforts to statistically determine the implied price of selected characteristics of housing. The price of a house is, in fact, the total of all prices paid for all aspects of the unit: its size, its amenities, its age, and so on. Hedonic studies relate the total price to the amount of each of the major characteristics in the unit and derive the price which the buyer has paid for each of those attributes. Ferri and Kain and Quigley offer examples of this kind of analysis of real estate values.

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