Comparative Investment Performance of Common Stock and Real Estate

by Moshe Ben-Horim

In a recent article, Michael S. Young suggested a methodology for comparing common stock and real estate investment returns. His proposal deserves several comments.

We shall start with Young's proposal for a measure of investment returns. He suggests to make "the simplifying assumptions that the starting value is the purchase price of the asset and that the income generated adds dollar-to-dollar to the value, while money to cover operating losses decreases the value." He recognizes the fact that the resulting measured returns are naive but says that ". . . we will take comfort in Professor Milton Friedman's comments: 'the relevant question to ask about the assumptions of a theory is not whether they are descriptively realistic, for they never are, but whether they are sufficiently good approximations for the purpose in hand. And this question can be answered only by seeing whether the theory works, which means whether it yields sufficiently accurate predictions.'" With naive assumptions, it is hard to see how we can "take comfort" in Friedman's comments when we have not yet seen whether or not the measure suggested by Young really "works."

As a measure of central tendency, Young suggests the geometric mean return on assets, which equals the *n*th root of the product of *n* wealth relatives minus 1.00. He says that whether the wealth relatives used are monthly, quarterly, or annual is a matter of "convenience, accuracy and availability." The fact is, however, that as long as the average is taken over a given period, the choice between monthly, quarterly, annual, or other wealth relative will yield the very same results. To illustrate, suppose nine end-of-quarter wealth levels are

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This article is a critique of "Comparative Investment Performance: Common Stocks Versus Real Estate—A Proposal on Methodology" by Michael S. Young, which appeared in the Summer 1977 edition (Vol. 2, No. 1) of *Real Estate Issues*.

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denoted by W_0, W_1, . . . W_7, W_8. The respective quarterly wealth relatives (WR) will be as follows:

$$WR_1 = \frac{W_1}{W_0}$$

$$WR_2 = \frac{W_2}{W_1}$$

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$$WR_8 = \frac{W_8}{W_7}$$

and the quarterly geometric mean return is given by:

$$1) \qquad \tilde{R} = \sqrt[8]{(WR_1)\ (WR_2)\ .\ .\ .\ .\ .\ .\ .\ (WR_8)} - 1.00 = \sqrt[8]{\frac{W_1}{W_0}} - 1.00$$

As we see, the intermediate wealth relatives do *not* add any accuracy to the calculation since $\bar{R}$ is a function of the wealth at the beginning and at the end of the entire period and the *number* of intermediate sub-periods in between. As an example, the monthly geometric mean return in the above example is $\sqrt[24]{\frac{W_8}{W_0}} - 1.00$, and the information on wealth levels of end of months for the period will add nothing to that measure.

Let us turn now to the risk measure and the measure of diversification. Young recommends using the beta coefficient in combination with the correlation coefficient; he advocates total reliance on these two measures and abandonment of traditional policies, particularly the self-imposed constraints of a maximum percentage of investment allowed in any one company or industry. Young demonstrated—using a hypothetical example—that the increased proportion of investment in real estate could reduce portfolio risk.

I do not wish to argue that Young's hypothetical example could not have a real life counterpart. But I do want to examine some potential problems with the approach he advocates.

Consider a portfolio consisting of a real estate investment yielding return $\tilde{R}_i$ and the "market portfolio" yielding return $\tilde{R}_m$. Denoting the proportions of investment by z_i and z_m respectively ($z_i + z_m = 1$) we may write the return on the portfolio ($\tilde{R}_p$) as follows:

$$2) \quad \tilde{R}_p = z_i \tilde{R}_i + z_m \tilde{R}_m$$

The expected return and variance of the portfolio are given by:

$$3) \quad \bar{R}_p = z_i \bar{R}_i + z_m \bar{R}_m$$

$$4) \quad \sigma_p^2 = z_i^2 \sigma_i^2 + z_m^2 \sigma_m^2 + 2 z_i z_m \operatorname{cov}(\tilde{R}_i, \tilde{R}_m)$$

where a bar over a random variable denotes expected value and where σ_i^2, and σ_m^2 and σ_p^2 are the variances of $\tilde{R}_i$, $\tilde{R}_m$ and $\tilde{R}_p$ respectively, and cov ($\tilde{R}_i$, $\tilde{R}_m$) is the covariance between $\tilde{R}_i$ and $\tilde{R}_m$. The *marginal contribution* of asset i to the portfolio variance is given by: [1]

$$5) \quad \frac{d\sigma_p^2}{dz_i} = 2\left[z_i(\sigma_i^2 - 2\beta_i\sigma_m^2) + \beta_i\sigma_m^2 - \sigma_m^2(1 - z_i)\right]$$

When z_i is very close to zero, the asset's own variance (σ_i^2) is a negligible element in the marginal contribution to the portfolio risk. As z_i rises, the variance of $\tilde{R}_i$ becomes more and more important in measuring the marginal contribution of asset i to the riskiness of the portfolio. It seems unjustified then, to recommend total reliance on β as a measure of risk on one hand and to allow z_i to rise with no restriction on the other. The above expression for marginal risk (i.e., $d\sigma_p^2 / dz_i$ is an improvement over β as a risk measure of those cases.

Other problems with the beta coefficient are related to its estimation. As we know, true beta coefficients are unobservable. We can only obtain *estimates* of true beta coefficients, and those are subject to sampling errors. Furthermore, even regardless of the sampling errors, there exists the question of the stability of beta: is next period beta going to be the same as today's beta?

Several stock market studies[2] indicate that the beta coefficient is not really stable, and thus the ex-ante beta coefficient would have been subject to uncertainty even if we knew the the ex-post beta with certainty. The sampling errors of the beta measures of real estate investments as well as their stability over time have not as yet been evaluated and thus it seems somewhat premature to make investment policies so dependent on quantities whose qualities are still unknown.

The best protection against sampling errors and uncertainty of other types is diversification. Thus the self-imposed constraints on maximum investment in any one asset company or industry should still make a lot of sense for portfolio managers.

1. In taking the derivative in equation 5 we made use of the following relationship:

$$\beta_i = \frac{\text{cov}(\tilde{R}_i, \tilde{R}_m)}{\sigma_m^2} \quad \text{so that } \text{cov}(\tilde{R}_i, \tilde{R}_m) = \beta_i\sigma_m^2$$

2. See Marshall E. Blume, "On the Assessment of Risk," *Journal of Finance* (March, 1971) and "Betas and Their Regression Tendencies," *Journal of Finance* (June, 1975); Stuart L. Meyers, "The Stationarity Problem in the Use of the Market Model of Security Price Behavior," *Accounting Review* (April, 1973), pp. 318-322.

REPLY

by Michael S. Young

Without going into an elaborate discussion of all the issues raised by Professor Ben-Horim in his Critique, let me say that we are not in disagreement with regard to the possible instabiilty or unpredictability of "beta" as a measure of risk or the occasional usefulness of marginal analysis. His argument is not wrong; we are simply talking about different things. He is delving into areas beyond the rudimentary presentation I made, and into regions of academic theory and research in which debate especially with regard to beta estimation and stability is currently raging.

In other areas of Professor Ben-Horim's comment he appears to have misinterpreted my intent or my English. For instance, I suggested that both betas *and* correlation coefficients be used to specify investment policy for holders of portfolios. In his discussion following the marginal contribution formula, he has lost sight of that which he earlier recognized. However, irrespective of the correlation coefficient restriction in the model, those who wish to try the computation will quickly discover that successive additions of an asset whose beta is lower than that of the portfolio to which it is added will not consume the entire portfolio but will reach a point of equilibrium unless constrained earlier by physical limitations.

Perhaps I got a little ahead of myself when I said that the choice of the periodicity of wealth relatives is a "matter of convenience, accuracy, and availability." Obviously, the geometric mean does not depend upon the intermediate wealth relatives but other computations such as the variance clearly do. In a paper addressed to a wide, generally non-academic audience, there is never the opportunity to go into all the minute details of proof.

Professor Ben-Horim's closing paragraph gives me reason to believe that the myth that diversification is enforced by specifying maximum percentages of a portfolio that should be invested in various assets will persist. Specifying percentages or quotes will *never* ensure diversification. Professor Ben-Horim and doubtless many others are guilty of this misunderstanding.

My article was intended to stimulate further discussion of advanced analytical techniques that might be applied to real estate. As a profession, real estate lags decades behind finance and economics. I had hoped that some institutional holder of real estate assets such as the insurance companies who maintain real estate portfolios for pension funds would submit or subject their portfolio to the kind of analysis necessary to substantiate or refute the proposition that real estate assets behave like common stock assets in such a way that useful comparisons could be made. Unfortunately no one has taken up the challenge.

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