CAPITALIZATION RATES, DISCOUNT RATES AND REASONABLENESS

by D. Richard Wincott, CRE,
Kevin A. Hoover and
Terry V. Grissom, CRE

In today's real estate markets, a tremendous emphasis is placed on the income capitalization approach to value, primarily the direct capitalization and discounted cash flow techniques. As a result, there seems to be continuing discussions regarding the relationship between the capitalization rate (R_O) and yield rate (Y_O) employed in the respective analyses. This relationship is generally stated in the equation $R_O = Y_O - CR$, where CR represents the constant ratio change in income and value.

This formula is perhaps the most misunderstood, overused and oversimplified property model. While some professionals swear by it, others disregard it as being completely invalid and not applicable in the real world. This article presents a practical analysis of the relationship between R_O and Y_O by addressing the inherent problems in the $R_O = Y_O - CR$ formula when applied to day-to-day analyses.

In general, there are two assumptions inherent in the $R_O = Y_O - CR$ formula that many overlook. First, this property model assumes that the capitalization rate and the yield rate are being applied to essentially the same income stream. In other words, the derivation of the income estimates in the two techniques must be the same. In practice, however, investors typically capitalized stabilized net operating income prior to capital cost deductions, while discounting the cash flow estimate after accounting for such costs as tenant improvement allowances and leasing commissions. Consequently, an adjustment to the property model is required.

The second assumption inherent in the model is that income and value grow at the same rate over the assumed holding period, and that the growth occurs on a constant ratio basis. Yet in the discounted cash flow models used by appraisers and investors, the growth in income and the growth in value often differ due to differences in the going-in and terminal capitalization rates as well as deductions for cost of sale in calculating the reversion estimate.

D. Richard Wincott, CRE, MAI, *is partner and chief appraiser for Price Waterhouse LLP's national Real Estate Valuation/Advisory Services Group. Wincott previously has published articles in* Real Estate Issues *and* The Appraisal Journal.

Kevin A. Hoover, MAI, CCIM, *is an appraiser in the General Account Division of Equitable Real Estate Investment Management, Inc., in Atlanta, Georgia. He is a member of the Appraisal Institute and the Commercial Investment Real Estate Institute.*

Terry V. Grissom, CRE, MAI, Ph.D., *is an associate professor of real estate at Georgia State University, Atlanta, School of Business Administration. His 20 years of real estate experience includes consulting, appraising and development, and he is also experienced in institutional investment research and portfolio analysis.*

Given these discrepancies between the inherent assumptions in the $R_O = Y_O - CR$ model and the practical application of the discounted cash flow models, modifications to the property model are required in order to accurately reflect the relationship between R_O and Y_O.

Simple Model

The following scenario illustrates the $R_O = Y_O - CR$ property model. This simple scenario is based on a year one income estimate of $1,000 and a compound growth rate of 4.0 percent. The reversionary value at the end of the 10 year holding period is estimated by capitalizing the 11th year income estimate at 10.0 percent, and the total present value of the income stream is derived using a discount rate of 14.0 percent. The cash flow estimates and present value calculations are summarized in Exhibit I.

EXHIBIT I

Growth Rate	4.00%
Discount Rate	14.00%
Terminal Capitalization Rate	10.00%

Period	Income	Present Value Factor @ 14.0%	Present Value
1	$1,000.00	0.877193	$877.19
2	1,040.00	0.769468	800.25
3	1,081.60	0.674972	730.05
4	1,124.86	0.592080	666.01
5	1,169.86	0.519369	607.59
6	1,216.65	0.455587	554.29
7	1,265.32	0.399637	505.67
8	1,315.93	0.350559	461.31
9	1,368.57	0.307508	420.85
10	1,423.31	0.269744	383.93
11	1,480.24		
Reversion	14,802	0.269744	3,992.87
Total Present Value			$10,000
Implied Capitalization Rate $1,000 / $10,000 =			10.00%
$Y_O - CR = R_O$ 14.0% − 4.0% =			10.00%

As the data indicates, the total present value approximates $10,000, resulting in an implied capitalization rate (R_O) of 10.0 percent ($1,000 ÷ $10,000). This model reflects the $R_O = Y_O - CR$ property model in its simplest form, as $14.0\% - 4.0\% = 10.0\%$. However, note that the capitalization and yield rates are applied to the same income stream, and the CRs for both income and value are equal.

Net Operating Income Vs. Cash Flow

The first problem to be addressed results from capitalizing net operating income while discounting cash flow after an allowance for capital costs. These deductions typically include such costs as tenant improvement allowances, leasing commissions and reserves for replacements. Given this difference, the $R_O = Y_O = CR$ model must be adjusted.

Consider Exhibit II, which again reflects a net income of $1,000, escalating at 4.0 percent over a 10 year holding period. However, a deduction is made for capital costs reflecting average tenant improvement allowances and leasing commissions. This deduction equates to $50.00 in year one, and also escalates at 4.0 percent over the holding period. The resulting cash flow is discounted at the yield rate of 14.0 percent.

The reversion again is calculated by capitalizing the 11th year net operating income; however, the terminal capitalization rate was adjusted to 10.5263 (10.53) percent so that the constant ratio change in property value would equal 4.0 percent, commensurate with the change in income.

As the data indicates, the total present value of the income stream approximates $9,500, resulting in an implied capitalization rate (R_O) of 10.53 percent. However, Y_O (14.0 percent) − CR (4.0 percent) equals 10.00 percent. The discrepancy between the implied capitalization rate of 10.53 percent and the rate implied by the property model of 10.00 percent results from the capitalization analysis employing the net income while the discounted cash flow analysis applies to the cash flow.

The implied capitalization rate of 10.00 percent can be adjusted for the differences in the income estimates by dividing the implied R_O by the ratio of average cash flow to net operating income. The adjustment to the formula is summarized:

$$(Y_O - CR) / (1 - \text{Capital Cost Ratio}) = RO$$

Where the Capital Cost Ratio equals the average ratio of capital expenses to net operating income

$$(14.0\% - 4.0\%) / (1 - 0.05) = 10.53\%$$

As indicated, the adjusted rate is equivalent to the implied capitalization rate derived by dividing the net income ($1,000) by the total value indication of $9,500. In practice, derivation of the capital cost ratio can be difficult, since capital deductions seldom occur on a straight line basis. Rather, the deductions typically fluctuate with various occurrences such as tenant rollover. Consequently, the ratio must be selected that reflects the average relationship between the cash flow and net income estimates. The timing of these costs must also be

EXHIBIT II

Growth Rate	4.00%
Discount Rate	14.00%
Terminal Capitalization Rate	10.53%

Period	Income	Capital Costs	Cash Flow	Present Value Factor @ 14.0%	Present Value
1	$1,000.00	$50.00	$ 950.00	0.877193	$833.33
2	1,040.00	52.00	$ 988.00	0.769468	760.23
3	1,081.60	54.08	$1,027.52	0.674972	693.55
4	1,124.86	56.24	$1,068.62	0.592080	632.71
5	1,169.86	58.49	$1,111.37	0.519369	577.21
6	1,216.65	60.83	$1,155.82	0.455587	526.58
7	1,265.32	63.27	$1,202.05	0.399637	480.39
8	1,315.93	65.80	$1,250.14	0.350559	438.25
9	1,368.57	68.43	$1,300.14	0.307508	399.80
10	1,423.31	71.17	$1,352.15	0.269744	364.73
11	1,480.24	74.01	$1,406.23		
Reversion	14,062			0.269744	3,793.24
Total Present Value					$9,500
Implied Capitalization Rate					
	$1,000 / $9,500 =				10.53%
$Y_O - CR = R_O$					
	14.0% − 4.0% =				10.00%

considered, as reflected by such factors as the average remaining lease term for existing tenants.

Differences In Income And Value Growth

Most would agree that we seldom see cash flow models in which the growth in income and the growth in value over a 10 year holding period are equal. The differences in the growth rates can be caused by a number of factors that may include differences between the going-in and terminal capitalization rates, deductions for costs of sale in the reversion calculation and deductions for anticipated capital expenditures at the reversion. Consider Exhibit III which employs an income estimate of $1,000 growing at 4.0 percent over the 10 year period, commensurate with the initial simple scenario. However, the reversionary value is calculated using a capitalization rate of 11.0 percent.

Based on a yield rate of 14.0 percent, the total present value of the income stream approximates $9,637.01, resulting in an implied capitalization rate of 10.38 percent. In this model, the $R_O = Y_O = CR$ model is difficult to apply, because the constant ratio change in income approximates 4.0 percent, while the constant ratio change in value approximates 3.40 percent, with the difference resulting from the higher terminal capitalization rate.

The discrepancy between the income and value CRs is exacerbated by current applications in the discounted cash flow analysis. Analysts typically

EXHIBIT III

Growth Rate	4.00%
Discount Rate	14.00%
Terminal Capitalization Rate	11.00%

Period	Income	Present Value Factor @ 14.0%	Present Value
1	$1,000.00	0.877193	$877.19
2	1,040.00	0.769468	800.25
3	1,081.60	0.674972	730.05
4	1,124.86	0.592080	666.01
5	1,169.86	0.519369	607.59
6	1,216.65	0.455587	554.29
7	1,265.32	0.399637	505.67
8	1,315.93	0.350559	461.31
9	1,368.57	0.307508	420.85
10	1,423.31	0.269744	383.93
11	1,480.24		
Reversion	13,457	0.269744	3,629.88
Total Present Value			$9,637
Implied Capitalization Rate			
	$1,000 / $6,637 =		10.38%
$Y_O - CR = R_O$			
	14.0% − 4.0% =		10.00%

make a deduction for costs of sale in calculating the reversion, but no such deduction is included in the direct capitalization analysis given the derivation of R_O. Further, it has become common practice to make a deduction in the reversion for capital items affecting the property at the time. These applications tend to widen the disparity between the CRs of the income stream and the value.

One method for adjusting the property model involves weighting the income and value CRs based on the percentage of total present value represented by the income stream and reversion. In this case, the present value of the income stream approximates 62.33 percent of the total present value, with the present value of the reversion approximating 37.67 percent of the total present value. Weighting the income and value CRs based on these percentages produces a weighted CR of 3.77 percent (4.00 percent × 62.33 percent) + (3.40 percent × 37.67 percent). The adjusted model is summarized:

Income CR =	4.00%
Value CR =	3.40%
Present Value of Income as a Percentage of Total Present Value	62.33%
Present Value of Reversion as a Percentage of Total Present Value	37.67%
Weighted CR	3.77%
Y_O – Weighted CR =	R_O
14.0% – 3.77% =	10.23%

This model implies an R_O of 10.23 percent, which is a close approximation of the implied capitalization rate of 10.38 percent. However, the model is not exactly accurate, and the variance will increase as the differences between the income and value CRs increase.

The weighted CR adjustment is technically invalid because the change in property value is not recognized in the discounting process on an annual basis, but rather in one lump sum at the end of the holding period. For example, assume three identical properties each reflecting current values of $10,000. Property A's value increases 10 percent in year one and remains flat for the remaining nine years of the 10 year holding period. Property B's value is flat for the first nine years of the holding period, and escalates 10 percent in the 10th year. Property C's value increases by one percent per year on a straight line basis over the 10 year holding period. In each case, the value at the reversion approximates $11,000, and in the discounted cash flow model, no value difference would be recognized since the proceeds to the owner from increases in value are not assumed to be received until the property is sold at the end of the holding period.

The Ellwood formula shown here can be used to addresss this discrepancy.

$$R_O = [Y_E - M(Y_E + P*1/S_n*R_M) - D*1/S_n] / K$$

The inapplicability of the Ellwood formula in this case is that it employs equity yield rates as opposed to property yield rates and considers the effect of financing. Since properties are typically analyzed on an unleveraged basis, the formula does not appear to be applicable in this instance. However, by eliminating the middle part of the numerator of the formula which deals with the mortgage financing, the Y_E in essence becomes a property yield rate, Y_O, as reflected in the following formula.

$$R_O = [Y_O - (D*1/S_n] / K$$

Where $K = \{1 - [(1+C)^n / S^n]\} / (Y - C)*A_n$
K = *Income Adjustment Factor*
D = *Total Property Value Change*
$1/S_n$ = *Sinking Fund Factor*
C = *Constant Ratio Change in Income*
S_n = *Future Value Factor*
A_n = *Present Value Factor of an Annuity*

Employing this formula allows the change in income to be addressed on a constant ratio basis and adjusted using the K factor calculation, while the change in the property value is addressed on a total basis and adjusted for using a sinking fund factor at the property yield rate. The following summarizes the calculations based on the previous model.

$$R_O = [Y_O - (D*1/S_n] / K$$

D =	39.64%
$1/S_n$ *(10 yrs @ 14%)* =	0.0517135
C =	4.00%
S_n *(10 yrs @ 14%)* =	3.7072213
A_n *(10 yrs @ 14%)* =	5.2161156
K = $\{1 - [(1 + 4.0\%)_{10} / 3.7072213]\} / (14.0\% - 4.0\%)*5.2161156$ =	1.1516487
R_O = $[14.0\% - (39.64\%*0.0517135)] / 1.1516487$ =	10.38%

The property model results in an implied R_O of 10.38 percent, exactly equaling the capitalization rate derived by dividing the net income of $1,000 by the total present value of $9,673.01.

Real World Application

Having addressed the two primary problems with the $R_O = Y_O - CR$ formula, the two revised models can be combined as shown below and applied to actual property scenarios.

$$R_O = \{[Y_O - (D*1/S_n)] / K\} / (1 - \text{Capital Cost Ratio})$$

In order to demonstrate the validity of this analysis, we have presented the actual income estimates for a

EXHIBIT IV

Discount Rate	12.00%
Terminal Capitalization Rate	8.50%
Costs of Sale	2.00%

Period	Income	Capital Costs	Cash Flow	Present Value Factor @ 12.0%	Present Value
1	$ 674,700.00	$13,900.00	$660,800.00	0.892857	$590,000.00
2	709,800.00	16,800.00	693,000.00	0.797194	552,455.36
3	721,500.00	22,300.00	699,200.00	0.711780	497,676.75
4	768,400.00	10,100.00	758,300.00	0.635518	481,913.36
5	785,600.00	20,700.00	764,900.00	0.567427	434,024.80
6	820,700.00	45,100.00	775,600.00	0.506631	392,943.10
7	863,900.00	9,200.00	854,700.00	0.452349	386,622.87
8	904,500.00	22,900.00	881,600.00	0.403883	356,063.45
9	925,300.00	24,600.00	900,700.00	0.360610	324,801.45
10	965,200.00	18,700.00	946,500.00	0.321973	304,747.67
11	1,005,900.00	24,800.00	981,100.00		
Reversion	11,597,435			0.321973	3,734,063.78
Total Present Value					$8,055,313
Implied Capitalization Rate $674,700 / $8,055,313 =					8.38%
Net Income CR =					4.0745%
Total Value Change =					43.9725%
Average Capital Cost Ratio = (Σ of Capital Costs / Σ of NOI)					2.51%

$$R_O = \{[Y_O - (D*1/S_N)] / K\} / (1 - \text{Capital Cost Ratio})$$

*K = Factor [1 − (1 + 4.07%)^{10} / 3.1058482] / (12% − 4.07%) * 5.6502230*	*1.1611577*
D = Total Property Value Change	*43.9725%*
$1/S_n$ = Sinking Fund Factor (10 yrs. @ 12%)	*0.0569842*
C = Constant Ratio Change in Income	*4.0745%*
S^n = Future Value Factor (10 yrs. @ 12%)	*3.1058482*
A_n = Present Value Factor of an Annuity (10 yrs. @ 12%)	*5.6502230*

$$R_O = \{[12.0\% - (43.97\% * 0.0569842)] / 1.1611577\} / (1 - 0.0252)$$

$$R_O = 8.39\%$$

major retail facility along with the assumptions used in the discounted cash flow analysis. This data is presented in Exhibit IV.

The model indicates an R_O of 8.39%, which is essentially equal to the implied R_O of 8.38%. By adjusting for the differences between net income and cash flow, as well as the differences in the income and value growth, the property model accurately depicts the relationship between R_O and Y_O.

Conclusion

All investment properties are unique and reflect a broad range of characteristics that impact potential income and therefore impact value. While we are not suggesting these dynamic investments be "put in a box," by use of a simple formula, we have concluded that there is a definite relationship between the appropriate Y_O and R_O for a given property. Understanding that relationship is essential in the process of selecting the appropriate rates, the key to understanding the relationship lies in an accurate analysis of the income characteristics that drive the direct capitalization and discounted cash flow analyses.