VALIDATION OF BASIC VALUATION MODELS: A MULTI-FAMILY HOUSING EXAMPLE

Mathematical models used to value income-producing property, such as the income capitalization approach, sales comparison approach, and estimation of expenses, are adequate reflections of the market.

by Phillip S. Mitchell and Gary L. Bernes

There are a number of simple mathematical models used in the valuation of income-producing property. These include the various income multipliers and the overall rate (OAR) which are associated with the income capitalization approach.[1] Similar multipliers are used in the sales comparison approach to value.[2] Expenses often are estimated on a per dwelling unit or per net rentable square foot basis or as a percentage of gross income or effective gross income. All of these mathematical models are directly proportional, almost the simplest models we can imagine.[3] How good are these simple models? They are certainly intuitively appealing and appear to conform with the realities of the market, at least to a first order of approximation over the relevant range of most real estate transactions.

The objective of the study reported in this article was to test and compare the simple, proportional models used in income property valuation. The testing and comparison used a basic statistical technique (multiple linear regression) on a good-quality, relatively homogeneous sample of 30 multi-family transactions in suburban collar areas in the Atlanta, Georgia, metropolitan area for the years 1984 through 1988. Although not large by statistical standards, the sample was of high quality.

The Database

Each record (transaction) included the number and type of units (DU) for the apartment complex, the annual gross rental income (GR), other income (OI), gross income (GI), vacancy and collection loss (VC), effective gross income (EGI), expenses (EXP), net operating income (NOI) and units by type. In addition, the actual or contract selling price (SP), the year built (YR) and the net rentable area (SF) were included. From these, a number of measures were derived, including age at sale (AGE), gross income multiplier (GIM), gross rent multiplier (GRM), effective gross income multiplier (EGIM), net income multiplier (NIM) and expenses per square foot of net rentable area (ESF), rooms (ERM), units (EDU) and bedrooms (EBD). The selling price also was analyzed as a ratio of square footage of net rentable area (SPSF), rooms (SPRM), units (SPDU) and bedrooms (SPBD).

In this analysis, all figures for income, expenses and selling price were rounded and reported in thousands of constant dollars.[4] In cases where the apartment complex was too new to have an extensive income history, the projected figures for the transaction were used.

Phillip S. Mitchell is a real estate consultant associated with Real Estate Sciences, International, a Chicago-based consulting firm. He was a professor of business administration in the California State University System for a number of years before entering the real estate business full time.

Gary L. Bernes, president of Bernes & Company of Atlanta, Georgia, has 15 years of commercial real estate experience, specializing in appraising apartments and hotels. He is a member of the International Society of Hospitality Consultants.

TABLE 1

Descriptive Statistics for Multi-Family Complex Database

Variable	Name	Mean	Minimum	Maximum	Coefficient of Variation
Number units	DU	244.30	64.00	490.00	0.53
Number rooms	RMS	1063.70	336.00	2398.00	0.06
Number Bedrooms	BDRMS	433.60	124.00	974.00	0.60
Number 1 bed, 1 bath	R11	92.60	0.00	406.00	n/a
Number 2 bed, 2 bath	R21	32.70	0.00	126.00	n/a
Number 2 bed, 2 bath	R22	103.90	0.00	350.00	n/a
Number 3 bed, 2 bath	R32	22.60	0.00	136.00	n/a
Net rentable area	SF	258.30	73.90	585.90	0.58
Year built	YR	82.10	75.00	88.00	0.05
Age at sale	AGE	4.60*	0.80*	14.00*	n/a
Selling price	SP	14459.00**	3888.00**	34181.00**	0.61
Gross rental income	GR	1819.50**	493.30**	3958.00**	0.55
Other income	OI	30.80**	4.20**	90.00**	0.69
Gross income	GI	1850.30**	500.60**	4048.00**	0.55
Vacancy and collection loss	VC	76.00**	12.50**	197.70**	0.66
Effective gross income	EGI	1774.30**	487.20**	3850.30**	0.55
Expenses	EXP	625.50**	234.50**	1335.00**	0.50
Net operating income	NOI	1148.80**	252.70**	2759.50**	0.59
Gross income multiplier	GIM	7.62	5.85	8.68	0.08
Gross rent multiplier	GRM	7.75	5.93	8.81	0.08
Effective gross income multiplier	EGIM	7.95	6.15	9.14	0.08
Net income multiplier	NIM	12.62	10.84	15.38	0.07
Expense / SF	ESF	2.62***	1.72***	4.21***	0.23
Expense / room	ERM	633.00***	430.00***	869.00***	0.21
Expense / unit	EDU	2656.00***	1981.00***	3664.00***	0.14
Expense / bedroom	EBD	1598.00***	1007.00***	2510.00***	0.27
Selling price / SF	SPSF	56.95***	42.07***	104.15***	0.24
Selling price / room	SPRM	57.80**	35.30**	73.00**	0.13
Selling price / unit	SPDU	13.70**	10.50**	21.10**	0.19
Selling price / bedroom	SPBD	34.60**	25.00**	60.60**	0.26

*Years
**Stated in thousands of dollars
***Stated in dollars

A summary of some descriptive statistics is reported in Table 1. These statistics include the mean, minimum, maximum and coefficient of variation for each measurement. For example, the smallest complex contained 64 units, while the largest contained 490 units. The mean for the variable "number of units" was 244.30. Annual gross rental income varied from $493,300 to $3,958,000 with a mean of $1,819,500, and so forth.

Valuation Model Tests

The database was analyzed using ordinary least squares regression techniques as embodied in the BMDP statistical software developed at the University of California, Los Angeles. A series of multiple linear regressions were run initially, with selling price (SP) as the dependent variable against the number of units (DU), age at sale (AGE) and a time (market conditions) variable (T), along with *one* of the commonly used value indicators listed in Table 1. These value indicators included gross rent (GR), gross income (GI), effective gross income (EGI), net operating income (NOI), area (SF), rooms (RMS), bedrooms (BDRMS) and number of units (DU).

The first of the analyses regressed selling price (SP) on gross income (GI), units (DU), age (AGE) and time (T).[5] This regression was highly significant and had a coefficient of determination of 99.7%, indicating the regression based on the four variables explained 99.7% of all the variation in the data. The units and age variables each had *student t* values of less than 1.00, while the time variable had a *student t* value of only 0.35. These values indicated that the time, unit and age variables had little explanatory power and could be dropped from the regression model. Most importantly, the intercept was not significant and was dropped by forcing a zero intercept term.

The regression was rerun using only the gross income variable (see Table 2). Similar analyses were performed to regress selling price against each of the remaining principal income-related explanatory variables and size, age at sale and time; these regressions produced similar results (see Table 2).

TABLE 2

Income Approach Model Results: Zero Intercept

Principal Explanatory Variable		Other Explanatory Variables			
	Coefficient (t Value)	Age at Sale	Time	Explained by Regression	Standard Error of Estimation
Gross income	7.99 (88.4)			99.6%	1,041
Gross rent	8.12 (85.4)			99.6	1,077
Effective gross income	8.32 (86.8)			99.6	1,060
Net operating income	12.63 (102.8)			99.7	896

Each of the regressions described in this article was examined for potential non-linearities and other statistical problems that would tend to render the regression results statistically unreliable. All of the regression results proved to be statistically reliable.

The regression results for the first four principal variables in Table 2—gross income, gross rent, effective gross income and net operating income—related to value indicators that typically would be used in the income capitalization approach to valuation.

The first line of the table presents the results of selling price regressed against gross income. None of the other independent variables appeared, in any combination, at a significant level. Well over 99% of the variation in the data was explained by this simple regression. Most importantly, since none of the other variables appeared at a statistically significant level, we concluded that the simple, directly proportional model that related selling price and gross income through a gross income multiplier was an entirely representative and valid approach to valuation based on this data set.

The next three lines in the table illustrate the results for three other regressions based on gross rent, effective gross income and net operating income. The results of these regressions were almost identical to the results of the first. Thus, taken as a whole, the income multiplier approach, in whatever form, appeared to be a valid technique for value estimation. In fact, for this data set, there was no important difference among the four income multiplier techniques.

TABLE 3

Comparable Sales Approach Results: Zero Intercept

Principal Explanatory Variable		Other Explanatory Variables				
	Coefficient (t Value)	Age at Sale	Time	Explained by Regression	Standard Error of Estimation	Increase in Standard Error
Dwelling units	62,160 (35.1)	−4,378 (−4.2)	4,789 (2.2)	98.9%	1,830	
	60,500 (40.1)			98.2	2,277	24%
Total N.R.A.	55,200 (33.8)	−12,594 (−5.9)	6,877 (6.3)	98.8	1,899	
	55,600 (29.9)			96.9	3,037	60%
Total rooms	13,380 (24.7)	−9,286 (−3.1)	4,955 (3.4)	97.9	2,574	
	13,500 (29.8)			96.8	3,042	18%
Total bedrooms	31,680 (20.3)	−11,790 (−3.4)	5,275 (3.0)	97.0	3,089	
	32,700 (24.9)			95.5	3,622	17%

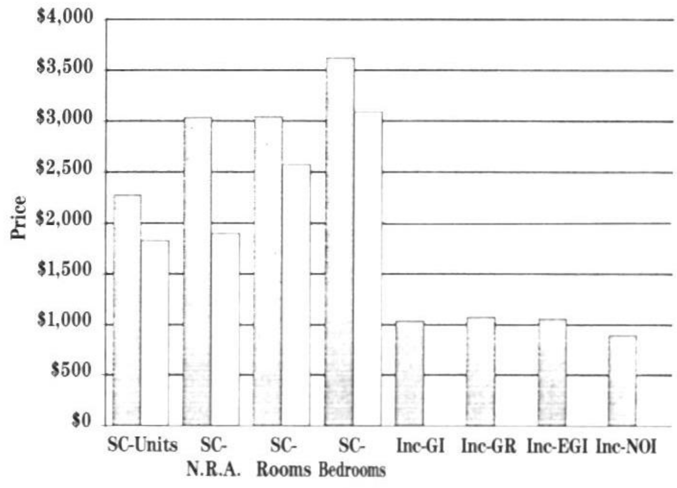
Table 3 includes regression models based on value indicators commonly associated with the comparable sales approach to valuation. These regressions provided some interesting results. For example, in the first regression shown, dwelling units (DU) proved, not surprisingly, to be the most important explanatory variable. However, both age at sale (AGE) and time (T) also had statistically significant *t* values and therefore played a statistically important, albeit small, role in the model. Thus, it appeared that the number of dwelling units was a good indicator of value (based on the second regression), even as a directly proportional unadjusted indicator. The same could be said for the other physical indicators—area, rooms and bedrooms—but not to the same degree.

The standard error of the regression on dwelling units alone was 24% greater than the standard error of the regression including time and age. The other increases in standard error were 60% for area, 18% for rooms and 17% for bedrooms. For all four regressions, these findings indicated a larger prediction error when the age and time variables were not used. A 17%, 18% or even 24% error could be adjusted for; a 60% higher prediction error was too high to be accepted graciously or adjusted for. Thus, the use of net rentable area, based on these data, could not be recommended. Further, adjustments for time and age at sale should be utilized in order to decrease error.

A final and very important conclusion was drawn by comparing the standard errors of estimate for the regressions in Table 2 and the regressions in Table 3. The standard errors for the sales comparison approach indicators, taken in their directly proportional form, ranged from 2,277 to 3,622 (Table 3), while those associated with the income capitalization approach ranged from 896 to 1,077 (Table 2). Thus, the indicators for the comparable sales approach had about two to three times the prediction error of the indicators for the income approach. This led us to the conclusion that, a priori, the income approach indicators provided value estimates that were considerably superior to those of the comparable sales approach. This result is shown graphically in Figure 1.

FIGURE 1

Standard Error in Sales Price Prediction Using Alternative Predictors



Note:
SC designates a variable used in the sales comparable approach;
Inc designates a variable used in the income approach.

Expense Model Tests

The simple direct proportionality models described above are often used to obtain valuation conclusions. Direct proportionality models are used to model expenses in the income approach to valuation. Specifically, expenses are estimated as a percentage of effective gross income, gross income or as an amount per dwelling unit; sometimes, they are estimated as an amount per room, bedroom or square foot.

We used simple regression analysis on our database as a means of analyzing and comparing these methods of expense estimation. The analysis proceeded in much the same way as described above and yielded similar results. Expense was regressed against a sequence of principal explanatory variables together with age at sale and time. The intercept in the regressions was never significant, indicating that directly proportional models were meaningful; thus, further regressions were forced to have a zero intercept.

Not surprisingly, time entered the regressions on physical indicators at a singificant level, but it was not significant for the income indicators. Age appeared at a significant level with the income indicators, but it was not significant for the physical indicators. However, the time (T) and age at sale (AGE) variables had relatively low impact. The existence of these variables in the regression indicated that appropriate adjustments should be made to the directly proportional models in order to reduce the prediction error increases that would otherwise occur. These increases can be seen in the final column of Table 4, along with the other regression results.

Table 4 also furnishes expected conclusions concerning the relative potential errors among the physical models and the income percentage models. First, the standard error of the regression for the simple, direct proportionality expense model based on dwelling units was much smaller than that for the rooms, area and bedroom models. Stated another way, these latter three models have standard errors which are 63%, 54% and 96% higher. Thus, among the physical expense models, it appears that expense per dwelling unit is superior.

Only two income models were considered, with the gross income regression carrying a 6% higher prediction error. This is hardly enough difference to make comfortable generalities, but the result does not differ from expectations. Thus, we should continue to favor the model based on effective gross income, since it corresponds most closely with experience and theory.

TABLE 4

Expense Regression Summary: Zero Intercept

Principal Explanatory Variable		Other Explanatory Variables			
	Coefficient (t Value)	Age at Sale	Time	Standard Error of Estimation	Increase in Standard Error
Dwelling units	2,361 (37.6)		17,869 (3.35)	66,035	
	2,513 (49.4)			76,789	16%
Total rooms	505 (23.6)		30,463 (3.79)	103,318	
	561 (30.1)			124,891	21%
Total N.R.A.	2.05 (28.8)		34,055 (5.21)	85,442	
	2.31 (31.9)			117,906	38%
Total bedrooms	1,192 (19.8)		37,197 (4.00)	121,936	
	1,355 (24.8)			150,218	23%
Gross income	30.6% (37.9)	−12,543 (4.5)		74,522	
	32.8% (39.5)			95,835	29%
Effective gross income	32.1% (40.7)	−12,053 (−4.6)		69,591	
	34.2% (41.8)			90,494	30%

Finally, the standard error for the expected gross income (EGI), model is 18% higher than the standard error for the dwelling unit (DU) model. This indicates that expenses are probably better predicted on a per dwelling unit basis rather than as a percentage of income.

- Among the *income multiplier models*, the net income multiplier (1/OAR) proved to be superior to the EGIM, GIM and GRM in terms of prediction error, but this superiority was not great. These multipliers appeared to need no adjustments for the time, age at sale or number of units in the complex; so all were valid in their simple, directly proportional form.

- The *per unit multipliers* associated with the sales approach to valuation were valid for the most part in their directly proportional form, but they could be adjusted for time and age at sale to reduce modeling (prediction) error. Dwelling units were the superior predictor, with total bedrooms and total rooms close behind. The total net rentable area had too high a standard error to be an adequate predictor.

- The *income multiplier models* generally were *superior* to the per unit multipliers by a factor of more than 2. Thus, it appeared that the income approach results should be favored, a priori, over the sales comparison approach results.

- All of the directly proportional *expense* models proved to be adequate, although all could benefit from adjustments for either time or age at sale to lower their prediction error. Estimation of expense based on effective gross income had a slightly lower prediction error than estimation as a percentage of gross income. However, estimation based on dwelling units had the smallest estimation error and, therefore, was the superior estimation method for this dataset.

None of the statistical results was surprising, and all generally supported the adequacy of the simple, directly proportional models that are commonly used by real estate professionals for valuation. These results probably generalize to larger, multi-family complexes under professional management.

NOTES

1. The OAR is equivalently and more conveniently represented as a net income multiplier, and it is properly grouped with other income multipliers.
2. Most of these models are applicable only to properties in which operations have been stabilized, although they still have usefulness as rules of thumb. When used as rules of thumb, these simple models require careful application due to comparability problems.
3. Their very simplicity is appealing in the extreme. It is difficult to imagine real estate professionals functioning without answering such questions as "what is the cap rate."
4. All nominal dollar data were converted to *constant* 1988 dollars based on the U.S. All-Urban Consumer Price Index.
5. Many more regressions were run than are reported here. For purposes of brevity, this exposition does not deal with the many relationships that did not prove to be significant.