EASY ACCESS COMPUTING CAPACITY MEANS INCREASED LIABILITY FOR REAL ESTATE APPRAISERS

Many of the financial formulas and techniques mandated by public sector regulators to calculate short-term loan charges are inaccurate and, in some cases, impose significant overcharges on consumers of credit.

by L. M. Farrell

While the number of financial applications of microcomputing technology has grown exponentially, various rule-of-thumb financial formulas still are used to calculate short-term loan charges. In many cases, these formulas are required by consumer protection legislation. For example, the 1974 Real Estate Settlement Procedures Act (RESPA), a federal statute enacted to protect home buyers, set guidelines for calculating interest due on settlement to cover the settlement period from closing to the first mortgage payment. According to *Settlement Costs and You*, an informational booklet prepared by the U.S. Department of Housing and Urban Development, these guidelines use a rule-of-thumb formula.[1] The formula, however, ignores the time value of money and overcharges home buyers for credit extended over the settlement period.

This article reviews the weaknesses of the principal rule-of-thumb formulas and suggests ways in which short-term loan costs can be calculated to eliminate these weaknesses.

Weaknesses Of Rule-of-Thumb Financial Formulas

Rule-of-thumb financial formulas have two significant weaknesses. The first is the use of a 360-day year instead of a 365-day year.[2] The second and conceptually more important weakness is the failure to recognize the time value of money.

The monthly compound interest tables that are found in most textbooks and used to find the compound value (of an annuity or a simple amount), the present value (of an annuity or a simple amount), the sinking fund factor and the mortgage constant, are inaccurate. Because they are based on a 360-day year composed of 12 months each consisting of 30 days, monthly compound interest tables overestimate the charges of interest per day.

Interest calculations, such as those contained in HUD's *Settlement Costs and You*, not only use a 360-day instead of a 365-day year,[3] but, more importantly, they neglect the time value of money. The booklet illustrates interest cost calculations using the following example:

> Suppose your settlement takes place on April 16 and your first regular monthly payment will be due June 1, to cover interest charges for the month of May. On the settlement date the lender will collect interest for the period from April 16 to May 1. If you borrow $30,000 at 9% interest, the interest item would be $112.50.

The rule-of-thumb formula used in this example does not consider the marginal contribution of interest that is earned on interest from one day to the next in reducing the amount of interest due. If the daily rate equivalent to the stated rate of 9% were used in the calculation of interest cost, interest would be $106.43. In the HUD example, the borrower would be overcharged $6.07 due to the bias in the rule-of-thumb formula.

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In the 1960s when interest rates and mortgage amounts were relatively low (in comparison to the 1980s), errors in calculation of short-term loan charges may not have been significant. However, given current interest rate levels, the amount of funds borrowed and the increased popularity of various repayment schedules (e.g., weekly payments), it may make sense, in some cases, to use more precise methods of calculation to determine credit costs, such as the time value of money method.

Mortgage Constants, Sinking Fund Factors And The Difference Between Nominal, Periodic And Effective Interest Rates

Microcomputers and hand-held calculators have made rule-of-thumb formulas, including financial tables, as obsolete as the engineer's slide rule. Now, equivalent interest rates can be calculated on an annual, semi-annual, quarterly, monthly, weekly or daily basis and allow quick comparisons of various what if scenarios. While continuous compounding is not in general use at the present time, it may become more important in the future to estimate the value of complex property rights, such as the appraisal of property rights acquired through an option to purchase land.[4]

Central to the calculation of equivalent interest rates is the distinction between nominal and effective annual interest rates. The *nominal annual interest rate* is the rate stated in the loan contract, and it is often accompanied by a statement of the frequency of compounding per annum.[5] The *periodic rate* is the rate of interest per compounding period, and it is calculated by dividing the nominal rate by the frequency of compounding. The *effective annual interest rate* is the actual rate of interest paid or earned after adjusting the nominal rate for the effect of compounding.[6] The effective rate is the rate at which funds could be loaned or borrowed to yield the same amount of interest if compounded only once a year rather than compounded at the frequency specified in the loan.[7]

The periodic debt service for a fully amortized loan can be decomposed into interest and principal repayment, using the relationship between the mortgage constant, the interest rate and the sinking fund factor.[8] The payment required to amortize a mortgage of \$M is the product $\$M \times \frac{1}{A_{\overline{n|}\,i}}$; in the first month, the interest payment is $\$M \times i$ and the sinking fund payment is $\$M \times \frac{1}{S_{\overline{n|}\,i}}$.

Calculations of equivalent interest rates can be made very quickly. For example, with a mortgage of \$90,000 that bears interest at 9% compounded monthly[9] and amortized over 25 years, the monthly payment (rounded off to the next cent) would be \$755.28.[10]

The interest payment in the first month would be (\$90,000) × (.0075), or \$675.00, and the repayment of principal would be \$755.28 – \$675.00, or \$80.28.

The principal repayment in the first month, which is sometimes called the *sinking fund payment,*[11] can be

TABLE 1

Effective Annual Interest Rate Based On A Given Nominal Rate Of 9% For Various Commonly Used Compounding Frequencies

Nominal Rate Per Annum %	Compounding Frequency N	Periodic[1] Rate %	Number of Compounding Periods Per Annum N	Effective[2] Annual Rate %	[Effective[8] Annual Rate] – [Nominal Annual Rate] %	Annual[3] Mortgage Constant %	Annual[4] Sinking Fund Factor %	Total[5] Annual Payment \$	Interest[6] Payment in First Year \$	Principal[7] Repaid in First Year \$
9	Annually	9	1	9	0	10.18063	1.18063	9,162.57	8,100.00	1,062.57
9	Semi-annually	$\frac{1}{2}=4.5$	2	9.2025	.2025	10.34814	1.14564	9,313.33	8,282.25	1,031.08
9	Quarterly	$\frac{9}{4}=2.25$	4	9.3083	.3083	10.43603	1.12773	9,392.43	8,377.47	1,014.06
9	Monthly	$\frac{9}{12}=0.75$	12	9.3807	.3807	10.49633	1.11563	9,446.70	8,442.63	1,004.07
9	Weekly	$\frac{9}{52}=.1731$	52	9.4089	.4089	10.51985	1.11095	9,467.87	8,468.01	999.86
9	Daily	$\frac{9}{365}=.0247$	365	9.4162	.4162	10.52594	1.10974	9,473.35	8,474.58	998.77
9	Continuous Compounding	N/A	N/A	9.4174	.4174	10.52694	1.10954	9,474.25	8,475.66	998.59

NOTES

1. Periodic rate = (nominal rate) ÷ compounding frequency.
2. Effective annual rate = $(1 + \text{periodic rate})^n - 1 \times 100\%$.
3. With annual mortgage constant and sinking fund factor, assume a 25-year amortization and 25-year mortgage.
4. The annual mortgage constant = the effective annual rate + the sinking fund factor.
5. Total annual payment = annual mortgage constant × \$90,000.
6. Interest payment in first year = effective annual rate × \$90,000.
7. Principal repaid in first year = total annual payment – interest payment in first year.
8. Effective rate with continuous compound is e^{it}, where i = .09 and t = 1 year.

TABLE 2

Nominal Annual Interest Rate Equivalent To A Rate Of 9% Compounded Monthly
For Various Commonly Used Compounding Frequencies

Nominal[1],[4] Rate Per Annum %	Compounding Frequency	Periodic[2] Rate %	Number of Compounding Periods N	Annual[3] Effective Rate Per Annum %	Difference Between Effective Annual Rate and Nominal Annual Rate %
9.3807	Annually	9.3807	1	9.3807	0
9.1704	Semiannually	4.5852	2	9.3807	.2103
9.0677	Quarterly	2.2669	4	9.3807	.3130
9.0000	Monthly	.7500	12	9.3807	.3807
8.9741	Weekly	.1726	52	9.3807	.4066
8.9675	Daily	.0246	365	9.3807	.4132
8.9664	Continuous Compounding	N/A	N/A	9.3807	.4143

NOTES

1. Nominal rate = periodic rate × compounding frequency.
2. Periodic rate = $\{(1 + \text{effective rate})^{1/n} - 1\} \times 100\%$, where the effective rate is expressed as a decimal.
3. The annual mortgage constant is .104963, based on a 25-year amortization period. It is composed of the annual interest rate of .093807 and a sinking fund factor of .011156 and remains constant with variations in the compounding frequency.
4. Nominal rate per annum = $\left\{1 + \begin{matrix}\text{effective}\\ \text{annual}\\ \text{rate}\end{matrix}\right\}$

calculated directly, using the sinking fund factor:

$$\text{Principal repayment in the first month} = \$90{,}000 \times \frac{1}{S_{\overline{300}|\,.75\%}} = \$90{,}000 \times (.000,892) = \$80.28$$

Relationship Between Periodic And Effective Mortgage Factors

Interest rate calculations can be generalized for any desired compounding frequency. Calculations of the effective annual rate based on a nominal rate of 9% per annum are presented in Table 1 for each of the most commonly used compounding frequencies. These calculations show that as the frequency of compounding increases, the effective rate increases but at a decreasing rate. This phenomenon indicates that larger increases in benefits to the lender (costs to the borrower) occur at lower frequencies and decrease as the frequency of compounding increases. For example, the increase in interest due to a shift from daily to continuous compounding is only $123 per $100,000 of debt. The benefits (costs) of more frequent compounding may not merit the cost of negotiation, therefore, for the average home buyer.

As the effective annual rate increases, the annual mortgage constant and the total annual payment increase, reflecting the increased interest payments (see Table 1). However, the total annual payment increases at a decreasing rate due to a declining sinking fund factor. The sinking fund factor decreases as the effective rate increases because at the higher reinvestment rate,[12] less of the annual payment has to be deposited in the sinking fund to amortize the loan.

The Equivalence Of Periodic And Effective Annual Mortgage Factors

The $90,000 mortgage in Table 1 can be used to illustrate the equivalence of the annualized periodic mortgage factors (the mortgage constant and the sinking fund factor) and the corresponding annual mortgage constant.

The annual mortgage constant (.104963) is the sum of the effective annual rate (.093807) and the sinking fund factor (.0111561). The total annual mortgage payment is (.104963) × $90,000, or $9,446.67. This payment includes an interest payment of .093807 × $90,000, or $8,442.63, and a principal repayment of .011156 × $90,000, or $1,004.04.

Suppose that, instead of making annual payments, the mortgage is amortized by a series of periodic payments calculated for various compounding frequencies (see column 2, Table 1) and the payments are deposited in a sinking fund at the periodic rate. The future value of these payments would equal the annual mortgage payment calculated at the effective rate.[13]

The Calculation Of Equivalent Periodic Rates

The Weekly Rate

The popularity of loan amortization schedules with more frequent compounding or more payments per years (e.g., the weekly amortization schedule), requires the calculation of the weekly equivalent rate and, for purposes of comparison, the periodic rates for compounding frequencies that are equivalent to a given annual effective rate.

Suppose that the borrower in the example above wanted to restructure the monthly mortgage amortization schedule to be repaid on a weekly basis. The lender would first calculate the weekly rate (r_w) which, when compounded 52 times a year, would be equivalent to a nominal rate of 9% per annum compounded monthly. This weekly rate would be 9.38% per annum.

Using r_w as equal to .1725[14] of 1% per week, the nominal rate[15] would be 8.974% per annum (rounded) compounded weekly. The weekly mortgage payment, or the product of the weekly mortgage constant and the initial loan amount, would be $173.80[16]; the sinking fund payment, or the product of the monthly sinking fund factor and the initial loan amount, would be $18.48; and the interest payment for the first week would be $155.32, the difference between the two.

The Daily Rate

For very large loans that are made for short periods of time measured in days, the lender would calculate the repayment schedule on a daily basis. The lender would calculate the daily rate (r_d) which, when compounded 365 times a year, would be equivalent to a nominal rate of 9% compounded monthly.

With the daily rate (r_d) equal to .0245685[17] of 1% per day, the daily payment on $90,000 would be $24.74 ($90.000 × the daily mortgage constant); the principal repayment on the first day would be $2.63 ($90,000 × the daily sinking fund factor); and the interest paid on the first day would be $22.11.

Both the lender and the borrower would be indifferent to 9.38% compounded annually, 9.00% compounded monthly, 8.974% compounded weekly or 8.967% compounded daily.

Nominal Annual Rates

Nominal rates with different compounding frequencies that are equivalent to 9% compounded monthly are presented in Table 2. The table shows that, as the frequency of compounding increases, the nominal rate declines but at a diminishing rate and illustrates the effect of the time value of money. Since the effective rate is constant, increases in compounding frequency mean that interest is calculated on interest more frequently, which thereby lowers the periodic and nominal rates. For the 25-year amortization period in the example above, the annual effective mortgage constant[18] (.104963) and the annual effective sinking fund factor (.011156) would remain constant with changes in the compounding frequency.

The Daily Interest-Only Mini-Loan

Daily interest-only mini-loans are used to provide interim bridge financing over very short periods of time,

TABLE 3

Interest Payment Overcharge[1] (In Dollars) For Various Effective Interest Rates And Mortgage Amounts

Mortgage Amount (dollars)	Effective Interest Rate %															
	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%	13.00%	14.00%	15.00%	16.00%	17.00%	18.00%	19.00%	20.00%
27,500[2]	2.10	2.82	3.64	4.55	5.56	6.66	7.85	9.12	10.49	11.94	13.47	15.09	16.78	18.56	20.41	22.34
30,000	2.29	3.08	3.97	4.97	6.07	7.26	8.56	9.95	11.44	13.02	14.69	16.46	18.31	20.25	22.27	24.38
40,000	3.05	4.10	5.29	6.62	8.09	9.69	11.41	13.27	15.26	17.36	19.59	21.94	24.41	26.99	29.69	32.50
50,000	3.81	5.13	6.62	8.28	10.11	12.11	14.27	16.59	19.07	21.70	24.49	27.43	30.51	33.74	37.11	40.63
60,000	4.57	6.15	7.94	9.93	12.13	14.53	17.12	19.91	22.88	26.05	29.39	32.91	36.62	40.49	44.54	48.75
67,500[3]	5.15	6.92	8.93	11.17	13.65	16.34	19.26	22.40	25.74	29.30	33.06	37.03	41.19	45.55	50.10	54.85
70,000	5.34	7.18	9.26	11.59	14.15	16.95	19.97	23.23	26.70	30.39	34.29	38.40	42.72	47.24	51.96	56.88
80,000	6.10	8.20	10.58	13.24	16.17	19.37	22.83	26.54	30.51	34.73	39.19	43.89	48.82	53.99	59.38	65.00
90,000	6.86	9.23	11.91	14.90	18.20	21.79	25.68	29.86	34.33	39.07	44.08	49.37	54.92	60.74	66.81	73.13
100,000	7.62	10.25	13.23	16.55	20.22	24.21	28.54	33.18	38.14	43.41	48.98	54.86	61.03	67.49	74.23	81.25
110,000	8.39	11.28	14.55	18.21	22.24	26.63	31.39	36.50	41.95	47.75	53.88	60.34	67.13	74.23	81.65	89.38
120,000	9.15	12.30	15.88	19.87	24.26	29.06	34.24	39.82	45.77	52.09	58.78	65.83	73.23	80.98	89.07	97.50
130,000	9.91	13.33	17.20	21.52	26.28	31.48	37.10	43.13	49.58	56.43	63.68	71.31	79.33	87.73	96.50	105.63
140,000	10.67	14.35	18.52	23.18	28.30	33.90	39.95	46.45	53.39	60.77	68.58	76.80	85.44	94.48	103.92	113.75
150,000	11.44	15.38	19.85	24.83	30.33	36.32	42.80	49.77	57.21	65.11	73.47	82.29	91.54	101.23	111.34	121.88
160,000	12.20	16.40	21.17	26.49	32.35	38.74	45.66	53.09	61.02	69.45	78.37	87.77	97.64	107.98	118.77	130.00
170,000	12.96	17.43	22.49	28.14	34.37	41.16	48.51	56.41	64.84	73.79	83.27	93.26	103.74	114.72	126.19	138.13
180,000	13.72	18.45	23.81	29.80	36.39	43.58	51.36	59.72	68.65	78.14	88.17	98.74	109.85	121.47	133.61	146.25
190,000	14.49	19.48	25.14	31.45	38.41	46.00	54.22	63.04	72.46	82.48	93.07	104.23	115.95	128.22	141.03	154.38
200,000	15.25	20.50	26.46	33.11	40.43	48.43	57.07	66.36	76.28	86.82	97.97	109.71	122.05	134.97	148.46	162.51

NOTES

1. Interest payment overcharge assumes an average settlement period of 15 days.
2. Mortgage of $27,500 is the current maximum VA guaranteed loan.
3. Mortgage of $67,500 is the current maximum FHA loan.

for example, when mortgage funds are loaned to home buyers for the interim between the settlement date and the date covered by the first mortgage payment. The interest due at the end of the bridge financing period is equal to:

$$\{(1 + r_d)^n - 1\} \times \text{(loan amount)}$$

where r_d is the daily rate and n is the number of days in the interim.

In the case of a mortgage of $90,000 at an effective rate of 9% per annum, if the average loan period is 15 days, the interest due at the end of the interim financing period would be $319.30.[19] The interest due for the same loan using the rule-of-thumb formula calculation suggested by HUD would be $337.50.[20] The borrower consequently would pay $18.20 more as a result of biases in the rule-of-thumb formula.

Estimation Of Unearned Interest From Consumer Overcharges

The amount by which consumers are overcharged[21] as a result of using the HUD rule-of-thumb formula is estimated in Table 3 for interest rates ranging from 5–20% on loan amounts from $27,500 to $200,000. At low interest rates, the interest overcharge ranges from $2.10 for a loan of $27,500 to $5.25 on a $200,000 loan. As interest rates and loan amounts increase, overcharges increase; the overcharge on a $200,000 loan at an interest rate of 20% is $162.51, not an insignificant sum.

While an exact estimate of the total amount borrowers are overcharged annually is difficult to derive, it very easily could range between $55–$75 million per year in the home market alone. In other areas of the economy, the amount of unearned interest paid by consumers could be significantly higher. To the extent that interest charges are tax deductible, the American taxpayer subsidizes a significant proportion of unearned interest overcharges paid by the unwary consumer.

Conclusions

Although microcomputers and hand-held calculators permit the immediate calculation of exact payable loan costs, HUD and possibly other federal regulatory agencies still require lenders to use rule-of-thumb formulas to calculate short-term loan costs. However, use of rule-of-thumb formulas often are costly to consumers. An analysis of the settlement period interest payment provision of RESPA indicates that home buyers are overcharged approximately $55 million per year due to the use of rule-of-thumb formulas. The method of computing short-term interest charges required by HUD and other public sector regulatory agencies should be replaced by the time value of money method of calculating interest. Such a change would produce significant nominal as well as real benefits for all concerned: it would please purists because the method is mathematically correct; it would allow consumers of financial services to realize significant savings in interest charges; and, to the extent that interest overcharges are tax deductible, the change would benefit American taxpayers by eliminating interest overcharges.

The net effect of introducing the time value of money methodology might be more beneficial than costly to lenders over the long term. Depending on competition and regulatory approval, lenders could offset any reduction in loan charges due to the use of the time value of money methodology by adjusting the nominal interest rate to reflect the decline in the effectives rate. The suggested modification would help eliminate misleading practices in the calculation of short-term credit charges, increase consumer satisfaction and simplify lenders' compliance with the truth-in-lending requirements established by the public sector.

APPENDIX 1
The Amortization Schedule

The amortization schedule, which shows how the periodic debt service is distributed between interest and principal repayments, can be derived using the mortgage constant and the sinking fund factor.

The First Month

In the first month of payment of a $90,000 mortgage, the monthly payment would be $755.28, $675.00 of which would be for interest and $80.28 for principal repayment.

The Second Month

In the second month, the principal repayment would be $80.28 plus the saving from interest that was not paid on the $80.28 repaid in the first month: ($80.28) × (.0075), or $0.60. The total principal repayment in the second month would be: ($80.28) + ($80.28) × (.0075) = $80.28 × (1.0075), or $80.88.

The interest payment would be (.0075) × ($90,000 − $80.28), or $674.40. Total interest paid at the end of the second month would be $675.00 + $674.40, or $1,349.40.

The total principal repaid at the end of the second month would be (principal repaid in first month) + (principal repaid in second month):

$= \$80.28 + \$80.28\,(1.0075)$
$= \$80.28 \times (1 + (1.0075))$
$= \$161.16$

The Third Month

In the third month, the principal repayment would be \$80.28 plus the saving from interest on the principal repayments of \$80.28 and \$80.88 made in the first two months.

The principal repayment in the third month would be: $\$80.28 + (\$80.28 + \$80.88) \times (.0075)$

which can be rewritten: $\$80.28 + (\$80.28 + \$80.28 + (.0075)\,(\$80.28)) \times (.0075)$

$= \$80.28 + 2(.0075)(\$80.28) + (.0075)^2 \times (\$80.28)$
$= \$80.28\,(1 + 2(.0075) + (.0075)^2$
$= \$80.28\,(1 + .0075)^2$
$= \$81.49$

The total principal repaid at the end of the third month would be:

$$\begin{pmatrix}\text{principal repaid}\\ \text{in first month}\end{pmatrix} + \begin{pmatrix}\text{principal repaid}\\ \text{in second month}\end{pmatrix} + \begin{pmatrix}\text{principal repaid}\\ \text{in third month}\end{pmatrix}$$

$= \$80.28 + \$80.28(1.0075) + \$80.28(1.0075)^2$
$= \$80.28\, S_{\overline{3|}\,.75\%}$
$= \$242.65$

where $S_{\overline{3|}\,.75\%}$ is the future value of an annuity of \$1 per period for three periods discounted at a periodic rate of .75 of 1% per period.

The interest paid in the third month would be:
$(\$90{,}000 - (\$80.28 + \$80.88)) \times (.0075) = \$673.79,$

which can be rewritten: (\$90,000 – (total principal repaid in months 1 and 2)) (.0075) = \$673.79.

The total interest paid at the end of the third month would be:
$\$675.00 + \$674.40 + \$673.79 = \$2{,}023.19.$

A simpler way to calculate the total interest paid at the end of the third month would be:

3 × (periodic payment) – (total principal repaid after three months)

$= 3 \times \$755.28 - \$80.28\, S_{\overline{3|}\,.75\%}$
$= \$2{,}265.84 - \$242.65$
$= \$2{,}023.19$

The K–th Month

For any month (K), the principal repaid is equal to $\$80.28 \times (1.0075)^{K-1}$ and the total amount of principal repaid at the end of the K-th month is $\$80.28\, S_{\overline{K|}\,.75\%}$ (Equation A-1).

The General Case

In general for a mortgage amount (M), the principal repaid in the K-th period would be:

$M \times 1/S_{\overline{n|}}\, i \times (1+i)^{K-1}$ (Equation A-1)

The total principal repaid at the end of the K-th period would be:

$M \times 1/S_{\overline{K|}\,i}) / (S_{\overline{n|}}\, i)$ (Equation A-2)

where i is the periodic interest rate and n is the number of compounding periods.

The interest paid in the K-th period would be:

$$\left\{\begin{pmatrix}\text{initial mortgage}\\ \text{amount}\end{pmatrix} - \begin{pmatrix}\text{total repayment of}\\ \text{principal, K - 1}\end{pmatrix}\right\} \times \begin{pmatrix}\text{the periodic}\\ \text{rate}\end{pmatrix}$$

which can be rewritten: $(M - M \times 1/S_{\overline{n|}}\, i \times S_{\overline{K-1|}\,i}) \times i$

$= M\,(1 - S_{\overline{K-1|}\,i}/S_{\overline{n|}}\, i) \times i$ (Equation A-3)

The total amount of interest paid at the end of the K-th period would be:

$k \times$ (periodic payment) $-$ (total principal repaid after K periods)

$= K \times M \times 1/A_{\overline{n|}}\, i - M \times S_{\overline{K|}\, i}/S_{\overline{n|}}\, i$

$= M\,(K/A_{\overline{n|}}\, i - S_{\overline{K|}\, i}/S_{\overline{n|}}\, i)$ (Equation A-4)

Initially, these results may seem complicated, but they can be calculated quite simply using a microcomputer or a calculator.

Example

Suppose that after five years, the borrower or lender wanted to calculate the following for a mortgage of $90,000 that bears interest at 9% compounded monthly and amortized over 25 years:

1. Periodic principal repayment
2. Total principal repaid
3. Periodic interest payment
4. Total interest repaid

In this case $K = 12 \times 5 = 60$, $M = \$90{,}000$, $n = 300$ months and $i = .75\%$ of 1%. The lender would substitute these values in Equations A-1 to A-4 as follows:

1. Principal repayment month 60
$= M \times (1/S_{\overline{n|}}\, i) \times (1 + i)^{K-1}$
$= \$90{,}000\,(1/S_{\overline{300|}\,.75\%}) \times (1.00750)^{59}$
$= \$124.76$

2. Total principal repaid
$= M \times (S_{\overline{K|}\, i}/S_{\overline{n|}}\, i)$
$= \$90{,}000 \times S_{\overline{60|}\,.75\%}/S_{\overline{300|}\,.75\%}$
$= \$6{,}055.05$

3. Interest payment month 60
$= M \times (1 - S_{\overline{K\text{-}1|}\, i}/S_{\overline{n|}}\, i) \times i$
$= \$90{,}000 \times (1 - S_{\overline{59|}\,.75}/S_{\overline{300|}\,.75\%}) \times 1.0075)$
$= \$630.52$

4. Total interest paid
$= M \times (K \times 1/A_{\overline{n|}}\, i - S_{\overline{K|}\, i}/S_{\overline{n|}}\, i)$
$= \$90{,}000 \times (60 \times 1/A_{\overline{300|}\,.75\%} - S_{\overline{60|}\,.75\%}/S_{\overline{300|}\,.75\%})$
$= \$39{,}261.75$

These results also can be calculated directly, using a preprogrammed financial calculator.

APPENDIX 2
Numerical Illustration Of The Equivalence Of The Periodic And Effective Annual Mortgage Factors

If the monthly mortgage payments in the example are deposited in a sinking fund which earns .75 of 1% per month, at the end of 12 months, the total would be: $\$755.28 \times S_{\overline{12|}\,.75\%} = \$9{,}446.67$, which is the equivalent annual mortgage payment required to amortize the mortgage over the 25-year amortization period at the effective rate of 9.38%.

Similarly, the principal repaid at the end of the first year would be equal to the accumulated value of the monthly principal repayments deposited in a sinking fund that earns .75 of 1% per month. By using Equation A-2 in Appendix 1 and substituting the given values, the calculation would be: $\$90{,}000 \times S_{\overline{12|}\,.75\%}/S_{\overline{300|}\,.75\%} = \$1{,}004.04$.

The interest paid in the first year would be the difference between the total annual amortization payment and the principal repaid in the first year: $9,446.67 − $1,004.04 = $8,442.63.

NOTES

1. *Settlement Costs and You, A HUD Guide For Homebuyers*, Washington, D.C.: U.S. Department of Housing and Urban Development, July, 1983, p. 21.

2. Weston, J. Fred and Copeland, Thomas E., *Managerial Finance*, 8th Ed. New York: The Dryden Press, 1981.

3. The implied interest calculation is:

$$\frac{30{,}000 \times .09}{360} \times 15 = \$112.50$$

4. Achour, Dominique and Brown, Robert L., "Appraising Land Options," *The Real Estate Appraiser and Analyst* (Summer, 1984).

5. American Institute of Real Estate Appraisers, *The Appraisal of Real Estate*, 8th Ed. Chicago: American Institute of Real Estate Appraisers, 1983.

6. The effective annual rate can be calculated as follows:

$$\left\{ \text{Effective annual rate (\%)} = (1 + \text{periodic rate})^m - 1 \right\} \times 100$$

where m is the frequency of compounding per annum.

7. Baxter, David; Hamilton, Stanley W. and Ulinder, Daniel D., *Foundations of Real Estate Financing*. Toronto, Canada: Butterworths, 1984.

8. The mortgage constant = interest rate + sinking fund factor:

$$\frac{1}{A_{\overline{n}|i}} = i + \frac{1}{S_{\overline{n}|i}}$$

where i is the periodic interest rate, m is the frequency of compounding, y is the mortgage amortization period in years and $n = m \times y$.

9. Effective annual rate $= \left\{\left(1 + \frac{.09}{12}\right)^{12} - 1\right\} \times 100$

$= 9.38\%$ where the periodic rate is .75 of 1%.

10. Monthly payment $= \$90{,}000 \times \frac{1}{A_{\overline{300}|.75\%}}$

$= \$90{,}000 \times (.008392)$

$= \$755.28$

11. The periodic amortization payment and the sinking fund payment can be used to calculate the interest paid and principal repaid as well as the respective cumulative amounts for any given future period in the amortization schedule. (See the amortization schedule in Appendix 1.)

12. Funds deposited in the sinking fund are assumed to be reinvested at the effective rate.

13. See Appendix 2.

14. $\{(1 + .09/12)^{12} - 1\} \times 100\% = \{(1 + r_w)^{52} - 1\} \times 100\%$.

15. Nominal rate = (periodic rate × (frequency of compounding).

16. Payments are rounded off to the next cent. The impact of rounding off on the final payment is not significant.

17. From

$$\left\{(1 + r_d)^{365} - 1\right\} \times 100\% = \left\{\left(1 + \frac{.09}{12}\right)^{12} - 1\right\} \times 100\%.$$

18. Note .093807 + .011156 = .104963, in Table 1 with monthly compounding.

19. $\{(1 + r_d)^{15} - 1\} \times \$90{,}000$

$= \{(1.0002361)^{15} - 1\} \times \$90{,}000$

$= \$319.30$ where $r_d = (1.09)^{1/365} - 1$.

20. (interest per day) × (number of days)

$= (.09 \times \$90{,}000/360) \times 15$

$= \$22.50$ interest per day × 15 days

$= \$337.50$

21. Calculated as of the end of the interim financing time period.