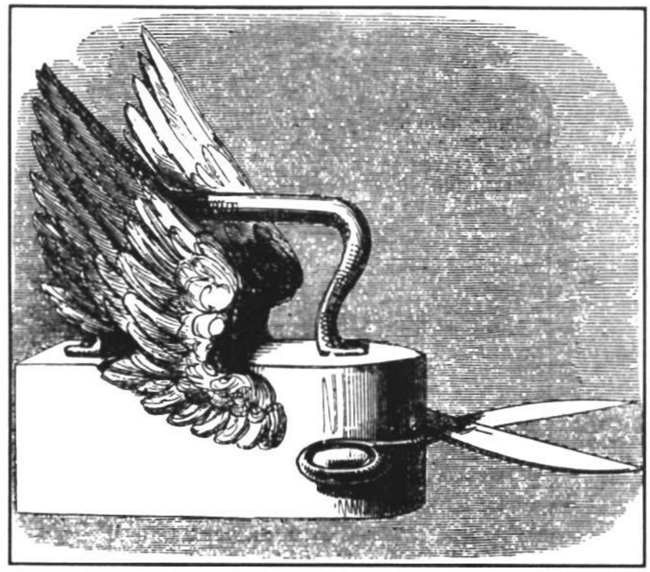
A CERTAINTY-EQUIVALENT APPROACH TO THE VALUATION OF RISKY REAL ESTATE INVESTMENTS

by Murad Antia, Steven D. Kapplin, and Richard Meyer

The valuation of real estate is one of the most significant activities in real property analysis. Purchase decisions, lending decisions, development and other decisions all rely upon valuation analysis. The quality and accuracy of an appraisal is therefore critical.

Recently concerns have been expressed about the distortion of values created by rapidly increasing rates of inflation, creative financing techniques, and changing project risks under uncertain economic conditions, and how they might impact on capitalization rates. The treatment of inflation, financial structure and project risk has been presented in various ways. This article presents a methodology for deriving value under conditions of risk. It will first present the background material on the basic theory behind the model. This will be followed by an illustration of two applications of the model; an example of a one-period project and an example of a multi-period project.



Background Material

This article derives principally from the Capital Asset Pricing Theory which was originally presented in the framework of corporate finance and investments. The development of the Capital Asset Pricing Model (CAPM) has been credited to Sharpe[1] and Lintner[2].

Murad Antia *is an assistant professor at the University of South Florida, College of Business, in Tampa, specializing in the areas of investment and portfolio analyses.*

Steven D. Kapplin *is an associate professor at the University of South Florida, specializing in real estate investments and valuation theory. He has published frequently in the* Real Estate Appraiser and Analyst, The Appraisal Journal *and* Real Estate Review.

Richard Meyer *is an associate professor at the University of South Florida in Tampa. He specializes in the areas of investment and portfolio analyses.*

The essence of the capital asset theory is the assumption that risky investments actually incorporate two types of risk—unsystematic risk and systematic risk. Unsystematic risk is viewed as being peculiar to the firm, such as risk of strikes or other labor disputes, inability to secure competitive prices on essential raw materials, or poor product marketing. Unsystematic risk is perceived as being different from firm to firm, so that investors, by careful selection, can devise investment portfolios which by virtue of diversification tend to cancel out unsystematic risk.

Proof of the ability of a portfolio of investments to reduce risk was presented by Markowitz[3]. Sharpe theorized that if investors could reduce or eliminate unsystematic risk through diversification, then they were not entitled to receive risk premiums to compensate for unsystematic risk. Therefore, only the systematic risk component was compensated for in the marketplace.

Systematic risk is perceived as an external risk factor, which is not peculiar to the firm and which affects all investments in the market, although not all to the same extent. Systematic risk is also often referred to as "market related risk". Sharpe, Lintner, and others showed that in a competitive market there was a linear relationship between investment returns and the correlation of investment risk to market risk. This latter concept of the relationship between investment and market returns became known as the investment's "Beta" coefficient—a risk measure which quantifies the riskiness of an investment relative to the market of all investments.

The capital asset theory provided the basis for developing the Capital Asset Pricing Model (CAPM), a quantitative model for estimating "risky" rates of returns for investments. Its form is presented below as equation (1):

$$k_j = R_f + [(r_{jm} \times s_j)/s_m] \times [E(R_m) - R_f] \quad (1)$$

where:

k_j = required return on asset j
R_f = risk-free rate of return
r_{jm} = correlation coefficient of returns between asset, j, and the market index, m
$E(R_m)$ = expected return on the market index
s_j and s_m = standard deviations of returns on asset, j, and the market index, m, respectively

The "Beta" risk measure is the term $(r_{jm} \times s_j)/s_m$ in equation (1). Thus, this equation can be restated as equation (2):

$$k_j' = R_f + B_j \times [E(R_m) - R_f] \quad (2)$$

The "market index" refers to some economic indicator to which individual investment returns are correlated. For most empirical research the market index commonly used is Standard and Poor's 500 Stocks Index. However, other indexes have been used including the Dow-Jones Industrial Stocks Index. Both are useful indexes of overall stock market performance, and have been used in most capital asset model tests because the model was developed primarily as a tool for deriving rates of returns on corporate securities. No such equivalent "market index" is widely available for real estate investment markets.

Since 1976 several articles in various publications have illustrated possible applications of the capital asset pricing model for deriving "risky" capitalization rates for investment real estate.[4] The usual approach for applying the CAPM involves preparing estimates of periodic rates of returns for individual projects and correlating them with the returns for some appropriate market index. Applying the model to real estate requires obtaining measures of project returns. The next section of this article will focus on this problem.

Deriving Correct Project Values

Let one assume a unique real estate investment which has a one period life. If this project were purchased for cost, C, and produced net cash flows of CF (which includes terminal proceeds), then the rate of return (ROI) for this project could be expressed as

$$R_j = CF_j/C_j - 1 \quad (3)$$

where:

R_j = expected return for project, j
CF_j = expected cash flow from project, j

If cost and value are equal, then R_j, the project's expected return, and k_j, the project's required return, are also equal. But appraisal theory teaches that cost and value are not necessarily synonymous. Value is usually defined in terms of highest and best use under normally competitive market conditions. When those conditions prevail, i.e., markets are in equilibrium, then cost and value tend to be the same. But when markets are in disequilibrium, cost and value may be quite different. When markets are in disequilibrium, the calculation of k_j based upon cost can become distorted. If k_j is calculated properly, then it is found as

$$R_j = CF_j/V_j - 1 \quad (4)$$

where:

V_j = project value
R_j = required project return

It should be obvious that if one uses equation (4), then a problem is presented. Value is a term in the denominator used for estimating project k_j. However, the purpose of estimating k_j was to estimate value. If cost and value were always assumed to be equal, then this problem is nonexistent. But since cost and value are not usually equal, then calculating a capitalization rate based upon cost will necessarily bias estimates of value. Depending upon the conditions of the market, this bias could be either upwards or downwards.

Since estimating value using capitalization rates requires knowing value in order to derive them, a sort of "Catch 22" is apparent. The following methodology presents a way of deriving market values which circumvents this dilemma. The methodology is unique in that it conforms with modern capital market theory and capital budgeting techniques, but does not require that k_j be known before value is estimated. This variation of the CAPM relies upon a valuation technique known as the "certainty equivalent" approach.[5] The methodology does not rely upon computing discount rates which may be biased, so it may have particular interest to the valuation profession.

Model Development

Previously, the basic elements of the CAPM were outlined and the model's format for deriving "risky" discount rates was described as equation (1):

$$k_j = R_f + [(r_{jm} \times s_j)/s_m] \times [E(R_m) - R_f]$$

In the above equation, k_j has been defined as the required return on risky asset j. Because the rate of return on a single period investment is simply the project's cash flows divided by the project's value, the project's rate of return can also be defined as

$$k_j = E(CF_j) / V_j - 1 \tag{5}$$

where:

$E(CF_j)$ = expected cash flow (which includes terminal value)

V_j = project value

The standard deviation of k_j as just defined would then be

$$s_j = s_{cf} / V_j \tag{6}$$

where:

s_{cf} = standard deviation of cash flows of project $_j$

If the preceding equations are substituted in the general model, equation (2), then one obtains

$$E(CF_j)/V_j - 1 = R_f + (r_{jm}/s_m) \times [E(R_m) - R_f] \times s_{cf}/V_j \tag{7}$$

or

$$V_j = \frac{E(CF_j) - (r_{jm}/s_m) \times s_{cf} \times [E(R_m) - R_f]}{1 + R_f} \tag{8}$$

This model conforms to the general assumptions of capital market theory and appraisal theory. All of the terms needed in equation (8) are readily available or can be easily computed. However, before proceeding, it is important to discuss the various terms of equation (8) and probable sources of data.

The risk-free rate, R_f, usually represents the return on a riskless investment of similar maturity to the investment under analysis. Generally, a government security is available which can act as the proxy for the risk-free rate. The term r_{jm}/s_m is a bit more difficult to derive. The numerator, r_{jm}, is the coefficient of correlation between the project returns and the return on the market index. Determining this correlation coefficient requires some additional data on the market index and another computational procedure similar to the one used to compute project expected cash flows and standard deviation.

The market index referred to here is itself a point of contention among researchers. Some of them think that when valuing real estate it is improper to use a non-real estate index like Standard and Poor's 500 Stock Index. They argue that the stock indexes reflect the performance of a completely different type of investment. Stocks are financial assets whose values are more sensitive than real estate to changing interest rates, inflation and general economic conditions. Further, they argue that the stock indexes tend to be more responsive to short-run effects. Real estate, on the other hand, is a real asset, long-term and generally producing more stable revenue streams. Stocks are divisible, while real estate must usually be purchased in its entirety.

Of course, these arguments are all worth considering. However, the availability of REITs, pension fund trusts, and limited partnerships would tend to make real estate a more divisible commodity. REITs, in particular, represent a financial asset more than a real asset. Nevertheless, one could argue that as long as all real estate investments were evaluated in comparison with one index, then there should exist a certain standard of comparison. Real estate returns could, then, be safely compared with the securities markets.[6,7]

This article does not purport to consider the relevant index, but rather to illustrate how that index would be used within the context of valuation using the certainty equivalent variation of the CAPM. To illustrate the application completely, some market index assumptions will be established so that the required data for the "certainty equivalent" model can be derived. The illustration will first present an example of a one-period project, then an example of the more general multi-period case.

An Illustration

Let one make the following project assumptions regarding a hypothetical real estate investment:

Probability of Occurrence	Net Operating Income
.10	\$ 50,000
.20	75,000
.30	100,000
.40	125,000

The calculation of E(NOI) or expected net operating income is:

E(NOI) =

.10 × 50,000 = 5,000
.20 × 75,000 = 15,000
.30 × 100,000 = 30,000
.40 × 125,000 = 50,000

E(NOI) = \$100,000 (Note: includes terminal proceeds because this is a one period project)

The expected net operating income for this hypothetical investment has been found to be \$100,000 per period. This was found by multiplying each assigned probability times the corresponding NOI associated with it, a method known as computing a "weighted" average. If all the probabilities had been equal (.25), then one could have used the "simple" average which is found by adding up individual NOIs and then dividing by the number of observations, which is four in this case.

Next, the standard deviation of the net operating income will be computed by using a table.

TABLE 1

Computation of Standard Deviation of NOI

(1) Probability	(2) NOI	(3) NOI_1-E(NOI)	(4) (Col. 3)^2	(5) Col. 1 × Col. 4
.10	\$ 50,000	\$−50,000	\$2,500,000,000	\$250,000,000
.20	75,000	−25,000	625,000,000	125,000,000
.30	100,000	0	0	0
.40	125,000	+25,000	625,000,000	250,000,000
			Variance =	\$625,000,000

Standard Deviation = $\sqrt{625{,}000{,}000}$
Standard Deviation (s_{noi}) = \$25,000

The mathematical procedure required to compute standard deviation first requires that one calculates variance. The standard deviation is then found as the square root of the variance. The mathematical formula is summarized as

$$\text{STD DEV} = \sqrt{\sum_{I=1}^{N} p_1 \times (NOI_1 - E(NOI)^2}$$

where:

STD DEV = standard deviation (s_{noi})

p_1 = probability of NOI_1

Σ = summation operator

N = number of occurrences, four in this example

The results for the hypothetical investment here indicate an expected NOI, E(NOI) = \$100,000 with a standard deviation of E(NOI), s_{noi} = \$25,000.

Next, a similar set of operations is performed in order to derive the characteristics of the market index.

Probability of Occurrence	Return on Market Index
.10	−.10
.20	.10
.30	.15
.40	.25

$$E(R_m) =$$

$$\begin{aligned} .10 \times -.10 &= -0.01 \\ .20 \times .10 &= 0.02 \\ .30 \times .15 &= 0.045 \\ .40 \times .25 &= 0.1 \\ E(R_m) &= 0.1555 \end{aligned}$$

In the preceding illustration there are some market index returns associated with probabilities corresponding to those used in the computations used for the hypothetical real estate project. The indicated market return is $E(R_m)$ = .1555, while the standard deviation of returns for the market is s_m = .10356. The following computations illustrate the determination of the correlation coefficient, r_{jm}:

TABLE 2

Computation of Standard Deviation of $E(R_m)$

(1) Probability	(2) (R_m)	(3) R_m-$E(R_m)$	(4) (Col. 3)²	(5) Col. 1 × Col. 4
.10	−.10	−.255	.065025	.0065025
.20	.10	−.055	.003025	.0006050
.30	.15	−.005	.000025	.0000075
.40	.25	.095	.009025	.0036100
			Variance =	0.010725

Standard Deviation = $\sqrt{.010725}$

Standard Deviation (s_m) = .10356

TABLE 3

Computation of Covariance Coefficient Between Project and Market

(1) Probability	(2) $(R_m$-$E(R_m))$	(3) $(CF_j$-$E(CF_j))$	(4) (Col. 2 × Col. 3)	(5) (Col. 1 × Col. 4)
.10	−.255	\$−50,000	\$12,750	\$1,275
.20	−.055	−25,000	1,375	275
.30	−.005	0	0	0
.40	.095	25,000	2,375	950
			Covariance Coefficient	=2,500

Correlation Coefficient r_{jm} is

$$\begin{aligned} r_{jm} &= Cov_{jm}/s_j \times s_m \\ &= 2{,}500/(.10356 \times 25{,}000) = .09656 \end{aligned}$$

The preceding result indicates that the real estate investment is correlated positively with the market index, which means that as the market index or return on the market increases, the return on the real estate investment will rise. When this paper was written, the prime rate was around .115 to .12. Since the simplifying assumption that this is a one period analysis has been made, .12 will be used as the assumed risk-free rate.

By substituting the known information into equation (8), the estimated project value, given the assumptions, is:

$$V_j = \frac{\$100{,}000 - [(.9656/.10356) \times \$25{,}000 \times (.1555 - .12)]}{(1 + .12)}$$

$$V_j = \$81{,}897$$

The value derived here is the correct "risky" value for the project, given the assumptions of the illustration. Having estimated value, one can also solve for the "risky" discount rate applicable to this or similar investments by using equation (7) to solve for k_j,

$$\begin{aligned} k_j &= [.12 + (.9656/.10356) \times (.1555 - .12) \times (\$25{,}000/\$81{,}897)] \\ &= .221 \end{aligned}$$

Because this is a one period investment, the correct "value" of the investment under conditions of risk is \$81,897. The "risky" discount rate applicable to this or similar projects is 22.1 percent. The next section will illustrate how this model can be extended from the simple one period example to the multi-period example. This latter illustration will be of specific application to most real estate projects since they usually involve holding periods greater than one year.

A Multi-Period Extension

The preceding illustration assumed a one-period investment. However, few real estate investments are single-period investments. The methodology discussed previously can also be applied to projects which have multiple cash flows occurring over more than one period. The derivation of the multi-period valuation problem is credited to Bogue and Roll.[8] It was adjusted for consistency with the capital asset pricing model framework by Fama.[9]

Equation (8) showed that a one-period investment could be valued as

$$V_i = \frac{E(CF_i) - (r_{jm}/s_m) \times s_{cf} \times [E(R_m) - R_f]}{1 + R_f}$$

For simplicity, one can reassign some of the symbols from the above equation as

$$r_{jm}/s_m \times s_{cf} = \frac{cov(E(CF),R_m) \times s_{cf}}{s_{cf} \times s_m \times s_m}$$

or

$$= \frac{cov[E(CF),R_m] \times [E(R_m) - R_f]}{s_m^2}$$

and define L as

$$L = [E(R_m) - R_f] / s_m^2 \tag{9}$$

Equation (8) can now be redefined by substituting equation (9) as

$$V_{t-1} = \frac{E(CF_t) - L_t \times cov(CF_t,R_{mt})}{1 + R_{ft}} \tag{10}$$

or

$$V_{t-1} = E(CF_t) \times [\frac{1 - L_t \times cov(e_t,R_{mt})}{1 + R_{ft}}] \tag{11}$$

where: $cov(e_t,R_{mt}) = cov(CF_t,R_{mt})/E(CF_t)$

Consider first a project which will produce a single cash flow more than one period into the future. The value in time period t−2 may be found as

$$V_{t-2} = E(V_{t-1}) \times [\frac{1 - L_{t-1} \times cov(e_{t-1},R_{mt-1})}{1 + R_{ft}}] \tag{12}$$

which, when equation (11) is substituted for the value of V_{t-1}, may be expressed as

$$V_{t-2} = E(CF_t) \times [\frac{1 - L_{t-1} \times cov(e_{t-1},R_{mt-1})}{1 + R_{ft}}] \times [\frac{1 - L_t \times cov(e_t,R_{mt})}{1 + R_{ft}}] \tag{13}$$

and the general valuation model derives as

$$V_0 = E(CF_t) \times \prod_{j=1}^{t} \times [\frac{1 - L_j \times cov(e_j,R_{mj})}{1 + R_{fj}}] \tag{14}$$

In order that this variation of the CAPM holds under conditions of uncertainty, only expected cash flow, $E(CF_t)$, can vary from period to period and be stochastic. The other parameters of the model, L_t, $cov(e_t,R_{mt})$, and R_{ft} may vary from period to period, but their values in each period must be known with certainty. If these conditions hold and L_t, cov (e_t,R_{mt}), and R_{ft} are constants, then the general valuation model may be reduced to the following form:

$$V_0 = E(CF_t) \times [\frac{1 - L \times cov(e,R_m)}{1 + R_f}]^t \tag{15}$$

which is equivalent to the constant risk-adjusted discount model typically used in financial and appraisal theory:

$$V_0 = E(CF_t) \times [1/(1+k)]^t \tag{16}$$

One may now assume investment in a project with multiple cash flows over a time horizon from t=1 to n and also make the typical assumption that each period's cash flow is equally as risky. In this case, each period's cash flow may differ, which would be typical in most investment feasibility analyses of net cash flows. Project value may be found as

$$V_0 = \sum_{t=1}^{n} E(CF_t) \times [\frac{1 - L \times cov(e,R_m)}{1 + R_f}]^t \tag{17}$$

Finally, one may assume that the cash flows expected in each period are constant. In other words, one is dealing with an annuity. This would be the typical assumption in usual income capitalization where the net operating income is assumed constant over the expected economic life of the property. In this case, the project's value is found by

$$V_0 = E(CF_0) \times \frac{\left[1 - [\frac{1 - L \times cov(e,R_m)}{1 + R_f}]^N\right] \times [1 - L \times cov(e,R_m)]}{R_f + L \times cov(e,R_m)} \tag{18}$$

If the project's cash flows co-vary positively with the market index, as is typical, and it has a perpetual life, its value would be found as

$$V_0 = \frac{E(CF) \times [1 - L \times cov(e,R_m)]}{R_f + L \times cov(e,R_m)} \tag{19}$$

The final section of this paper illustrates the usage of the preceding equations with the numeric data from the previously described example of the one-period investment.

A Multi-Period Illustration

In order to keep this illustration simplified, the data developed for the previously illustrated one-period investment will be used.

TABLE 4

Summary of Data for One-Period Investment

1. Expected cash flow (NOI)	\$100,000
2. Expected market return (index)	15.55%
3. $Cov(E(CF_t),R_{mt})$	2,500
4. $L_t = \frac{[E(R_m)-R_f]}{s_m^2}$	3.31012
5. R_f	12.00%

By using the information in Table 4, one can develop the solution by recognizing that this particular problem usually assumes that net operating income is constant over the holding period. This requires that one use equation (18) for the solution. Equation (18) is reproduced as

$$V_0 = E(CF_0) \times \frac{\left[1 - \left[\frac{1 - L \times cov(e,R_m)}{1 + R_f}\right]^N\right]}{R_f + L \times cov(e,R_m)} \times [1 - L \times cov(e,R_m)]$$

For the purpose of this illustration it is assumed that the holding period will be ten years, N=10. The basic computations are summarized as

$$\begin{aligned} L \times cov(e,R_m) &= L \times cov[E(CF),R_m] / E(CF) \\ &= 3.31012 \times 2{,}500 / 100{,}000 \\ &= .08275 \end{aligned}$$

therefore,

$$\begin{aligned} 1 - L \times cov(e,R_m) &= 1 - .08275 \\ &= .91725 \end{aligned}$$

substituting in equation (18),

$$V_0 = 100{,}000 \times \frac{\left[1 - \left[\frac{.91725}{1.12}\right]^{10}\right]}{.12 + .08275} \times .91725$$

$$\begin{aligned} &= 100{,}000 \times \frac{.86426}{.20275} \times .91725 \\ &= 100{,}000 \times 4.2627 \times .91725 \\ &= 100{,}000 \times 3.90997 \\ V_0 &= \$390{,}977 \text{ or rounded to } \$391{,}000 \end{aligned}$$

The valuation of the hypothetical property producing an expected net operating income of $100,000 (with standard deviation of $25,000) is found to be $391,000. One may recall that in the previous one-period illustration the risky discount rate was 22.1 percent. By substituting the risky discount rate in the standard annuity discount formula for the present value of an annuity, one finds that the present value annuity factor, PVIFA, is 3.91, thus yielding the same results as just described. The current illustration, however, demonstrates that it is possible to derive the risk-adjusted valuation of property or other income-producing investment without knowing the required discount rate in advance. The data required on the project and on the market are more easily derived than the required discount rate.

Summary And Conclusions

This paper has presented a variation of the Capital Asset Pricing Model (CAPM) using the certainty equivalent approach to derive an estimate of value for risky projects. The advantage of the methodology outlined here is that it does not require knowing or deriving a risky discount rate for solution. The technique only requires that the appraiser derive expected net operating income and standard deviation of net operating income, expected return and standard deviation for a market index. Using this basic information, an estimate of value can be derived for one-period investments or multi-period investments which have uneven or annuity cash flow patterns. The technique, then, permits appraisers to make valuation estimates on risky income properties without first having to derive or estimate required market discount rates for the properties under evaluation.

The possible disadvantages of such a methodology include lengthy computational procedures, lack of an acceptable market index (although this problem can be overcome by using the same index for all properties), and probable difficulty in communicating the logic of the method to clients. However, it is felt that such disadvantages are overcome by having a method of valuation which is not dependent upon estimating or deriving required risky discount rates through market analysis. Finally, the methodology is useful for valuing projects that are not financed, that is, cash equivalent value.

BIBLIOGRAPHY

1. Sharpe, William, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance,* (September 1964), 425-442.
2. Lintner, John, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *The Review of Economics and Statistics,* (February 1965), 13-37.
3. Markowitz, Harry, "Portfolio Selection," *Journal of Finance,* (March 1952), 77-91.
4. For a review of this literature see, "Institutional Real Estate Portfolios," Chapter 27, *Real Estate Investment: Strategy, Analysis, Decisions,* Stephen A. Pyhrr and James R. Cooper, Boston: Warren, Gorham and Lamont, 1982.
5. Robicheck, Alexander A. and Myers, Stewart C., *Optimal Financing Decisions,* Englewood Cliffs: Prentice-Hall, Inc. 1965, 80-83.
6. Hoag, James, "A Real Estate Return Index: Measurement of Risk and Return," *Proceedings of the Seminar on the Analysis of Security Prices,* University of Chicago, (May 1979), 223-259.
7. ______, "Towards Indices of Real Estate Value and Return," *Journal of Finance,* (May 1980), 569-580.
8. Bogue, M. C. and Roll, R. R., "Capital Budgeting of Risky Projects with 'Imperfect' Markets for Physical Capital," *Journal of Finance,* (May 1974), 601-613.
9. Fama, Eugene F., "Risk-Adjusted Discount Rates and Capital Budgeting Under Uncertainty," *Journal of Financial Economics,* (August 1977), 3-24.