THE REAL ESTATE INVESTMENT DECISION— A WEALTH MAXIMIZATION APPROACH

by C. F. Sirmans and Daniel E. Page

During the last decade real estate has been receiving widespread attention as an attractive investment, probably due to the relative price changes it has undergone as well as the increasing awareness of its tax shelter benefits.[1] Substantial advances have been made in the theory of the investment decision; however, considerable debate has arisen over the best measure to use in ranking investment proposals.[2] Ellwood, Strung, Friedman, and Messner and Findlay believe that the internal rate of return (IRR) or some variation is the best measure to use. Wendt and Cerf believe a net present value (NPV) model is best.

This paper develops an NPV model that will lead to maximization of the investor's current wealth, an objective consistent with the goals of a rational investor.[3] The model can be used by the real estate investor to determine: 1) the investment decision; and 2) the holding period that leads to wealth maximization. Sensitivity analysis will be performed to see how a wealth maximizing solution is affected by changing some of the impact variables.

Investment Decisions: Objectives And Criteria

The first step of the investment decision is to identify the investor's goals. Then appropriate criteria for reaching these goals are selected. Reasons for investing in real estate are: 1) investment security; 2) available cash flows; 3) financial leverage; 4) tax shelter benefits; 5) property value appreciation; 6) equity position; and 7) inflation hedge. The basic underlying objective is maximization of current wealth.[4]

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Measures of ranking investment proposals have been developed and are the internal rate of return (IRR) or some variation, and the net present value (NPV) method.

The IRR has received significant attention as the standard measure of return on equity investments in real estate.[5] The Ellwood and Inwood rates are in essence the IRR. Finance literature has long recognized the pitfalls of using the IRR as a measure of ranking investment proposals.[6] In 1955 Lorie and Savage pointed out the possibility of multiple IRRs when the cash flows have more than one sign change.[7] Also, maximizing the IRR will not lead to wealth maximization.[8] In recognition of the problems of the IRR method, other measures based on it were developed including the adjusted IRR and the Financial Management Rate of Return (FMRR).

The adjusted IRR, developed by Strung in recognition of the reinvestment rate assumptions of the IRR, is a modified IRR that allows for cash flows to be invested at some realistic reinvestment rate. A shortcoming of this measure is that it does not account for other problems such as multiple IRRs.

The FMRR was designed by Messner and Findlay in an attempt to account for all shortcomings of the IRR. This measure still introduces a bias in calculating the rate of return generated by a particular investment. The prespecified reinvestment rate of the FMRR may be no more appropriate than the IRR of the investment.

Given all the problems of the IRR, any variations still contain a bias. The NPV method avoids these problems, correctly discounts at the opportunity cost of funds, and is precisely the same thing as maximizing the investor's current wealth.

The Wealth Maximizing Model

The traditional model of real estate investment decision making can be written as follows:[9]

$$E = \sum_{t=1}^{n} \frac{NOI_t - A_t - I_t - T_t}{(1+r)^t} + \frac{SP_n - SE_n - GT_n}{(1+r)^n}$$

Where:

E = the present value of the equity
NOI_t = net operating income in period t
A_t = mortgage amortization in period t
I_t = interest paid on mortgage in period t
T_t = income taxes in period t
SP_n = sales price in period n
SE_n = selling expenses in period n
GT_n = capital-gains tax in period n
n = the expected holding period
r = the required after-tax rate of return

This model calculates the present value of the investor's expected after-tax cash flows from operations and sale given a specific holding period (n). If the NPV associated with a specific n is positive, the investment should be undertaken. Since real estate values fluctuate over time, the NPV calculated for a specific n may not be at a maximum. As stated earlier, a rational investor will try to maximize current wealth by selecting the holding period that maximizes net present value. Finding this holding period requires the calculation of the NPV for each year of the investment's economic life. The resale value of the investment in each year of its economic life also must be considered. Ignoring these resale values may lead to a suboptimal investment decision.

Extending the traditional model to allow for selling the investment in any year, and assuming that all cash flows are known with certainty, leads to the following model.[10]

$$\underset{n^*}{\text{MAX}}\ NPV_E = \sum_{t=1}^{n^*} \frac{ATCF_t}{(1+r)^t} + \frac{ATER_{n^*}}{(1+r)^{n^*}} - E_o$$

Where:

n^* = expected holding period
NPV_E = net present value of the equity investment
$ATCF_t$ = the after-tax cash flow in year n^*
$ATER_{n^*}$ = the after-tax equity reversion in year t
E_o = initial equity investment and all other variables as previously defined

The holding period of the investment is the period, n*, that will maximize the NPV of the equity investment. Steps to determine maximum n* NPV are as follows:

1. Set $n^* = 1, 2, 3, \ldots, n$, where n = the economic life of the investment
2. Compute $NPV_1, NPV_2, NPV_3, \ldots, NPV_n$
3. Select the n* that maximizes the NPV
4. If maximum $n^* > 0$, buy the investment and sell in the year that corresponds to the maximum n*

The use of the model is twofold: 1) to make the investment decision; and 2) to determine the holding period of the investment. To make the investment decision, the investor would choose a specific n*. If NPV associated with that n* were positive, the investment should be made. However, the NPV for a specific n* may not be at a maximum. By doing a complete enumeration over the economic life of the investment, the maximum NPV could be found.

Comparative Statics

The maximum n* NPV is a function of the after-tax cash flows from operations and reversion. Table 1 is an analysis of the comparative static changes in the holding period resulting from changes in key variables. By allowing only one variable to change while holding all other variables constant, the effect on the holding period can be observed. For instance, if rents were to increase (other things being equal), the holding period that maximizes the NPV would increase. An increase in operating expenses will result in the holding period decreasing.

TABLE 1

Comparative Static Changes
In Maximum n* NPV With Holding Period (n*)

Variable	Effect on Holding Period
Rent	$\frac{\partial n^*}{\partial rent} > 0$
Operating expenses	$\frac{\partial n^*}{\partial OE} < 0$
Property value	$\frac{\partial n^*}{\partial property\ value} > 0$
Interest rate (loan)	$\frac{\partial n^*}{\partial i} < 0$
Selling expenses	$\frac{\partial n^*}{\partial SE} < 0$
After-tax required rate of return	$\frac{\partial n^*}{\partial r} < 0$

Simulation

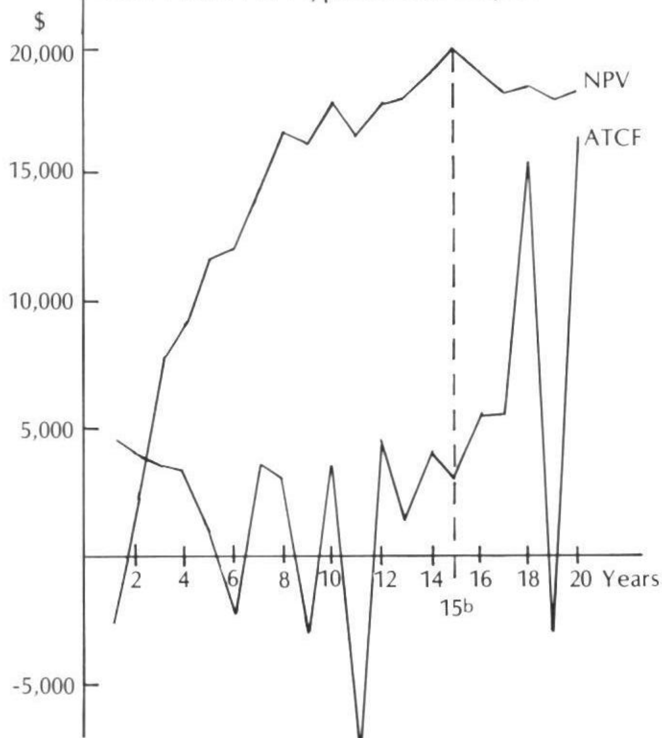
To illustrate the application of the model, a hypothetical project was proposed.[11] An investor considers purchasing a small office building with the following characteristics:

- Purchase price: $88,515
- Rent per month: $1,500
- Growth rate in rent per year: 4%
- Occupancy rate: 95%
- Property tax and insurance per month: $200
- Growth rate of taxes and insurance per year: 4%
- Operating expenses per month: $180
- Growth rate of operating expenses per year: 4%
- Depreciation basis: $78,115
- Depreciation method:[12] 150% declining balance; component method with replacement of assets as they reach the end of their useful life
- Amount borrowed: $70,812
- Length of loan: 15 years
- Expected appreciation of property value per year: 4%
- Selling expenses at time of sale: 10%
- Investor's tax rate: 50%
- Required after-tax return on equity: 12%

Table 2 lists the expected after-tax cash flows, after-tax equity reversion, the present value of the equity investment, and the NPV of the equity investment for 20 years. To make the investment decision, the investor would choose a specific holding period n*, and determine the NPV for that holding period. If the NPV is positive, the investment should be made. For

FIGURE

Cash Flows For Hypothetical Project[a]



a) See text for assumptions
b) Wealth maximizing holding period

TABLE 2

Cash Flows For Hypothetical Project[a]

Year	ATCF	ATER	PV	NPV[b]
1	$ 4,537	$ 12,617	$15,316	$-2,387
2	3,999	15,616	19,688	1,985
3	3,666	19,441	23,686	5,983
4	3,421	24,031	27,295	9,592
5	1,196	29,526	29,455	11,752
6	-2,077	35,674	29,722	12,019
7	3,732	41,770	32,232	14,529
8	3,535	48,721	34,442	16,739
9	-3,206	57,206	34,238	16,535
10	3,876	65,333	35,892	18,189
11	-7,622	75,628	34,407	16,704
12	4,519	84,845	35,603	17,900
13	1,114	95,601	35,990	18,287
14	4,143	107,102	36,844	19,141
15	3,441	119,831	37,541	19,838[c]
16	5,421	124,642	36,866	19,163
17	5,476	129,481	36,188	18,485
18	15,451	133,255	36,668	18,965
19	-3,077	141,028	35,356	17,653
20	16,489	145,353	35,760	18,057

a) See text for assumptions

b) $NPV_E = \sum_{t=1}^{n^*} \frac{ATCF_t}{(1+r)^t} + \frac{ATER_{n^*}}{(1+r)^{n^*}} - E_0;$

where $E_0 = \$17,703$

c) Wealth maximizing holding period

TABLE 3

Sensitivity Analysis

Rent per month	Wealth maximizing holding period
$1,250	10 years
1,500	15
1,750	15
2,000	20
2,280	20
Interest rate on loan	
.05	15 years
.07	15
.09	15
.11	15
.13	15
.15	15
.17	15
Growth rate in property value	
0%	15 years
2	15
4	15
6	15
8	15
10	18
12	20
After-tax required rate of return	
6%	20 years
8	20
10	18
12	15
14	10
16	8
18	8

example, if the investor chooses a holding period of 10 years, the NPV would be $18,189. Through the use of the model, the investment decision can be made. This holding period, however, does not maximize the investor's wealth. As seen in the table, NPV is maximized in the 15th year. According to the model, this would be the holding period. The NPV of the equity investment at the maximum would have a value of $19,838. This example (portrayed graphically in the Figure) illustrates why it is necessary to perform a complete enumeration over the investment's entire life.

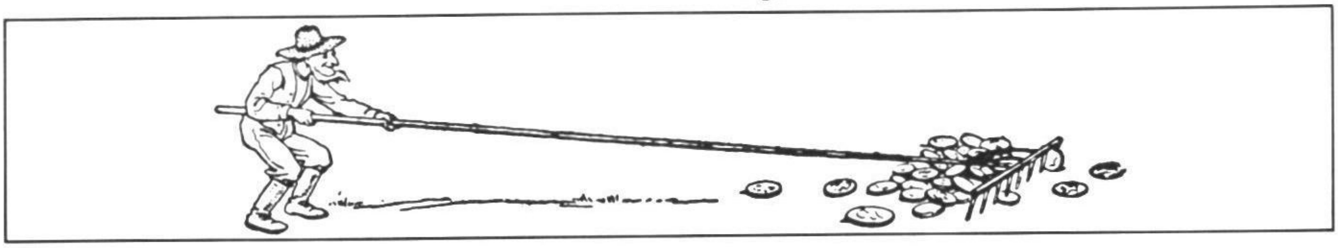
Although the holding period of this example was 15 years, all real estate investments do not have a holding period of this length. The assumptions of the investment determine the holding period.

Sensitivity analysis to examine the change in the holding period was performed on four key variables: 1) rent per month; 2) the interest rate on the loan; 3) the growth rate in property value; and 4) the investor's required after-tax rate of return. Table 3 lists the changes in the wealth maximizing holding period when the model was simulated over various ranges of these variables. As expected, the direction of the change in the holding period was the same as in the comparative static analysis in Table 1. The interest rate on the loan and the growth rate in property value had little effect on the maximum NPV.

Summary And Conclusions

A model was developed that could be used by a real estate investor to: 1) make the investment decision; and 2) determine the wealth maximizing holding period. The model is an NPV approach to wealth maximization and by using it the shortcomings of the popular IRR method are eliminated.

An example illustrated that complete enumeration over the life of the investment must be performed to determine the holding period that maximizes the investor's wealth. Sensitivity analysis showed how an optimal solution changed when key variables changed.



NOTES

1. A study conducted by Eugene Fama and G. William Schwert, "Asset Returns and Inflation," *Journal of Financial Economics* 5 (November 1977), 115-146, estimated the extent to which Treasury bills, Treasury bonds, Common stocks, private residential real estate, and labor income provided hedges against expected and unexpected inflation for the period of 1953-71. They found that real estate was a complete hedge against both types of inflation.

2. Techniques for the valuation of real estate investments range from rules of thumb — gross income multiplier (GIM), to discounted cash flow models — net present value (NPV), to computer simulation models. For a discussion of these techniques, see C. F. Sirmans and Austin J. Jaffe, *Real Estate Investment Handbook*, Prentice-Hall, 1981, Chapters 8 and 9, and Oakleigh J. Thorne, "Real Estate Financial Analysis—The State of the Art," *The Appraisal Journal* (January 1974), 7-37.

3. Eugene F. Fama and Merton H. Miller, *The Theory of Finance* (Illinois: Dryden Press, 1972, Chapter 1).

4. C. F. Sirmans and Austin J. Jaffe, *Real Estate Investment Handbook* (Prentice-Hall, 1981, Chapter 2).

5. Stephen D. Messner, Irving Schreiber, and Victor L. Lyon, *Marketing Investment Real Estate: Finance Taxation Techniques* (Illinois: The Realtors National Marketing Institute® of the National Association of Realtors®, 1975), 43.

6. For a review of the literature on the IRR, see Austin J. Jaffe, "Is There a 'New' Internal Rate of Return Literature?" *AURUEA Journal* (1977), 482-503.

7. Multiple IRRs are not the only problems of the IRR method. The IRR is also sensitive to the size of the initial outlay and the timing of the cash flows. Also, the reinvestment rate assumption of the IRR method may not be appropriate. For a discussion of the problems of the IRR, see Eugene F. Brigham, *Fundamentals of Financial Management* (Illinois: Dryden Press, 1978), 277-286.

8. Thomas E. Copeland and J. Fred Weston, *Financial Theory and Corporate Policy* (Addison-Wesley Publishing Company, 1979, Chapter 2).

9. Paul F. Wendt and Alan R. Cerf, *Real Estate Investment Analysis and Taxation* (New York: McGraw Hill, 1979 2nd Edition), 52-54.

10. The future cash flows in investment analysis are often uncertain. To allow for these random variables, the model can be written as:

$$\underset{n^*}{\text{MAX}}\ \widetilde{NPV}_E = \sum_{t=1}^{n^*} \frac{\widetilde{ATCF}_t}{(1+r)^t} + \frac{\widetilde{ATER}_{n^*}}{(1+r)^{n^*}} - E_o$$

Where: all variables as previously defined, except the tildes (~), indicate random variables

Assuming a discrete probability distribution could be specified for each cash flow, a finite set of cash flows would occur in each period. Expected values and variances could then be calculated. The cash flows are not independent from one year to the next. Cash flows in year t+1 would depend on events in period t. The model would basically be a one-period autocorrelation model. Each year the investor's forecast would improve concerning upcoming cash flows. Thus, the model would become a dynamic programming problem. Technically, the model would perform the same. The investor would select the holding period that maximized expected current wealth. Abandonment options under uncertainty are discussed more fully in Charles P. Bonini, "Capital Investment Under Uncertainty With Abandonment Options," *Journal of Financial and Quantitative Analysis* (March 1977), 39-54.

11. The cash flows for this example were calculated by a computer program called JMODL, which was developed by the Texas Real Estate Research Center. The basic assumptions of the program are: 1) cash flows can be estimated with certainty; 2) taxes are computed using the 1978 real estate tax laws with no provision made for minimum tax; and 3) any year that a negative cash flow is projected, the investor will borrow funds at some specified short-term interest rate. These borrowings are repaid with future positive cash flows.

12. The components are depreciated as follows. The land had a cost of $10,400 or 11.75% of the total cost of the investment.

Asset Description	Cost	Percent Of Total Cost	Economic Life
Basic structure	$51,254	57.90%	20 years
Electrical system	3,606	4.07	20
Plumbing	8,000	9.04	18
Roof	800	.90	15
Vacuum system	3,500	3.95	10
Light fixtures	1,157	1.31	8
Appliances	2,226	2.51	8
Carpet/vinyl	4,614	5.21	5
Draperies	1,000	1.13	5
Paint/stain	1,958	2.21	4
	$78,115	88.25%	

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