ALTERNATIVES FOR ASSESSING RISK IN REAL ESTATE INVESTMENTS

by Richard J. Curcio, James P. Gaines, and James R. Webb

The assessment of risk in real estate investing is being given more and more attention. In a comprehensive evaluation of previous empirical evidence on real estate returns, Roulac [1976] concluded that while real estate and stock market returns are comparable over time, for given levels of return, real estate generally is less variable and more predictable. Roulac attributes this lower risk level to the basic economic pattern of real property as compared to corporate enterprise, and also to the enhanced dispersion of results from common stock securities. Roulac's conclusion that real estate investments have relatively lower risk than securities is based on highly restrictive empirical studies which rely on the variance or a variance-related measure such as the standard deviation or coefficient of variation for the assessment of risk. Webb and Sirmans [1980] also use coefficient of variation.

The validity of the variance as a proper risk surrogate has been questioned.[1] Its principal limitation is that its reliability depends on the shape of the relevant distribution of returns. For symmetric distributions, the problem is somewhat reduced; however, the general existence of symmetry in investment portfolios has not been established.[2] For the pure equity non-security form of real estate investment returns, the existence of symmetry appears even more suspect because for a given wealth position real estate investing seems to provide less ability for diversification and more potential for large losses than investments in securities. Conclusions regarding the riskiness of real estate investment securities may have to await more extensive empirical studies.

Alternative risk measures and risk screening approaches that have received substantive attention in the investment literature and particularly in regard to securities are beta, semi-variance, skewness, kurtosis, and stochastic dominance. This study considers the applicability of established security risk measures as alternatives in assessing real estate investment risk.

Review Of Previous Studies

Much treatment of the risk in real estate investment

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has involved to some degree the use of intuitive techniques such as adjusting the discount rate to correspond with the riskiness of the investment [Shipp, 1970] or adjusting the anticipated returns downward to reflect their relative uncertainty (that is, using certainty equivalents) [Wiley, 1976]. Another popular approach to risk in real estate investing is sensitivity analysis [Farrell, 1969; Higgins & Cunningham, 1970]. While neither measuring nor adjusting for risk, sensitivity analysis does enable the identification of the critical variables underlying forecasts of return, and this allows for more effort to be allocated to obtaining greater accuracy in estimating these variables and implicitly producing more reliable return forecasts.

The use of probability distributions for evaluating risk in real estate investing is more explicit and promising than the above method.[3] Prominent empirical studies which employed probability distributions or distribution parameters for assessing risk in real estate are the works of Wendt and Wong [1965], Friedman [1970], and Robichek, Cohn and Pringle [1972].

Wendt and Wong compared the investment experience of 20 FHA-financed apartment houses with 76 randomly chosen industrial stocks. They used the coefficient of variation on the distribution of internal rates of return, and their results indicated considerably lower risk for real estate than for common stock investments.

Friedman attempted adaptation of the mathematical models used to analyze and select security portfolios to the evaluation and selection of real estate portfolios. Using samples of 50 properties from two sources[4] and 50 common stocks from the New York Stock Exchange, he generated efficient frontiers for each class of assets on a before and after-tax basis. His results indicated a lower level of risk associated with a specific rate of return for real estate than for common stocks. Computed as the variance of past yields, the variance was employed to measure risk. Because of a lack of market data, however, Friedman was forced for the real estate sample to assume a constant compound rate of growth over the sample period, 1963 to 1968, which may have resulted in underestimation of the total variance for the real estate investments.[5]

Robichek, Cohn and Pringle compared the investment merits of farm real estate with eleven alternative investment media. The coefficients of variation for the farm real estate were found to be considerably lower than the others which included the Standard and Poors Industrial Index.

These empirical results on the explicit measurement of risk in real estate may suggest that real estate investments are less risky than common stock. The results, however, were based on highly restrictive real estate samples and may have involved measurement errors. In addition, the studies relied largely on the variance or variance-related measures to evaluate risk. Deficiencies of the variance and related gauges cause questions about their soundness in assessing risk and the conclusions relating to risk in real estate investments.

Risk Measurement In Real Estate

Explicit measurement of investment risk has focused in general on the analysis of subjectively-derived probability distributions of investment return. In these analyses, quantitative risk surrogates typically are used to replace common verbal definitions of risk which do not yield easily to measurement. Usually these surrogates represent some measure of the dispersion of outcomes in the relevant probability distribution. As such, each is regarded as a gauge of the uncertainty characterizing this distribution and is thus considered a viable measure of the risk inherent in the asset or asset combination from which the particular distribution is derived.[6]

Not all the risk surrogates will yield identical assessments either on an absolute or relative basis. The assessment of risk, the relative ranking of the alternatives and the ultimate investment decisions may vary substantially depending on the choice of a risk surrogate.

The popular use of variance type measures to assess real estate risk is explained when one considers that most, if not all, of the current methods for treating real estate risk have drawn substantially from the Markowitz portfolio selection model. In adapting the Markowitz model, financial writers have tended to accept his mean-variance (E-V) criterion for choosing among risky alternatives even though no conclusive evidence supports the use of variance as the proper risk surrogate. Ease of computation and broad familiarity are its most often cited advantages and dependence on the configuration of the underlying distribution is its greatest limitation. Symmetry in the distribution of returns substantially alleviates the problem. For widely skewed distributions, consideration of the third and higher moments or other measures of dispersion such as the semi-variance may be more appropriate.

Systematic Risk

The capital asset pricing model (CAPM) defines the following equilibrium pricing relationship for securities:

$$E(R_i) = R_f + \beta_i\,[E(R_m) - R_f]$$

where E is the expectation operator, R_i and R_m the return on the ith security and the market portfolio, respectively, and R_f the risk-free rate of interest.[7] Beta (β_i) denotes the systematic risk or market sensitivity of the ith security and is expressed mathematically as:

$$\beta_i = \mathrm{Cov}(\widetilde{R}_i\widetilde{R}_m)/\mathrm{Var}\,\widetilde{R}_m$$

where the ($\sim$) denotes a random variable.

Estimates of β_i are typically determined from a time series regression of the following variant of the first equation:

$$R_{it} = \alpha_i + \hat{\beta}_i R_{mt} + \eta_{it}$$

where R_{it} and R_{mt} represent the realized values in period t, α_i and $\hat{\beta}_i$ the intercept and slope coefficients, respectively, and η_{it} is a random disturbance term with zero mean and zero intertemporal and intercompany covariance.

Limitations to the applicability of systematic risk in real estate become apparent when one examines the model's assumptions. The CAPM requires that all investors: 1) be single-period, risk-averse, expected utility maximizers; 2) have homogenous expectations about future returns for each asset; 3) be content to characterize assets on the basis of mean and variance of return; and 4) be able to borrow or lend as much as they like at the risk-free rate. The model also assumes no transaction costs or taxes, and requires each asset to be perfectly divisible.

Unlike securities, real estate investments tend to be large, indivisible, illiquid and highly leveraged. Real estate transaction costs are typically large; and mortgage interest rates generally are well above those of riskless instruments. Perhaps of greater significance, real estate markets appear to be relatively inefficient compared to security markets. Some evidence of nonrandomness in real estate price changes was found by Upson [1975]. Also, Roulac [1976] delineates a comprehensive list of explanations and reflects on the lack of quality and quantity of information among reasons for existing real estate market inefficiencies.

In addition, the CAPM requires an appropriate and adequate market index (not yet identified for real estate investments) and assumes that investors will diversify sufficiently to eliminate random or what is called unsystematic risk.[8] Friedman [1970, 1972] used the Sharpe [1963] diagonal model — a single index approach — to generate an efficient frontier of a sample of real estate assets. The index employed constituted an average of the Boeckh construction cost indexes for residences, apartments, hotels, commercial construction and factories and the American Appraisal Association Index. It appeared to be severely limited and produced the questionable result that real estate assets were less risky than common stocks, bonds and mortgages.[9]

Also, the relatively larger size (in terms of dollar cost) of individual real estate investments as well as the greater required involvement in operating management may limit the pure real estate investors' ability to diversify away unsystematic risk. For example, given an initial equity level of $150,000, a common stock investor could purchase (assuming no transaction costs) 100 shares of 20 different companies at $75 per share (approximately the current average price of a share of stock on the NYSE). For comparison and if one assumes no leverage, a real estate investor with the same initial equity could purchase only one $150,000 property, a somewhat conservative price for a typical income producing real estate investment.[10] If one considers leverage, the common stock investor using the current maximum allowable leverage of 50 percent could purchase 100 shares of approximately 40 companies. With the same 50 percent leverage, the real estate investor could only purchase two $150,000 properties. Even if one considered 70 to 80 percent leverage which is more typical for real estate investment, he still would be able to purchase only three or at most five properties.

Diversification (reduction of random risk) is assumed to increase with the number of investments in specific, unrelated assets included in the portfolio. Empirically, Evans and Archer [1968] found that for security investments, unsystematic risk approached zero with the inclusion of between 7 and 15 randomly selected securities. While comparable evidence does not exist for real estate, it appears that for a given initial equity position, removal of unsystematic risk will occur more readily with securities than for a pure real estate portfolio.[11]

A real estate equity investor, that is, one who purchases the actual property, requires a greater role in operating management than is necessary in securities investment. The equity investor may manage the property on his own or hire a professional. Either way, there may be a tendency to restrict the property investments to the same or proximate geographic region for practical and economic advantages. This may impede the opportunity for the important consideration in real estate investment of interregional diversification. Management specialization, that is, the condition that different expertise is required for managing various classes of properties, may tend to restrict the number of property types in the portfolio.

These characteristics of real estate investments and markets would seem to place heavy limitations on the application of the CAPM to direct equity real estate investing. Such limitations may be overcome in time, since there are indications that the real estate investment market is becoming less inefficient. Roulac [1976] points out that recent trends toward real estate in securities, the institutionalizing of the real estate investment business and increased government involvement in the control of land use, housing, transportation, and socio-economic patterns will enhance the overall efficiency of the real estate investment market.

The CAPM has demonstrated appealing tractability in securities investment applications. The ability to capture an investment's complete relevant risk composure in a single, highly comparable measure, the beta or market risk factor, has contributed to a relatively rapid and growing practical acceptance of the CAPM by investors and investment counselors.[12] Insofar as this tractability extends to real estate, it seems to justify the search for an appropriate real estate or multi-asset index and to stimulate efforts to overcome other obstacles to the application of the CAPM to real estate investing.

Semi-variance

The semi-variance is defined by:

$$S_h = E\{\mathrm{Min}[(R-h), 0]\}^2$$

where R is a random variable with a known probability distribution and h denotes a critical value against which actual values of R are compared. Relative to the variance, given by $\sigma^2 = E[R-E(R)]^2$, and which regards all extreme returns as undesirable, the semi-variance as a risk measure has the advantage of focusing on reduction of losses. For this reason Markowitz [1959] considered the semi-variance to be superior to the variance, although he opted for the latter because of its familiarity and ease of computation. Mao [1969] explored the merits of the semi-variance risk measure for the corporate capital investment case. His approach primarily involved a conceptual comparison of mean-variance (E-V) versus mean-semi-variance (E-S_h). He concluded "that the E-S_0 model is particularly useful in making capital budgeting decisions. In such instances, one is usually concerned with a relatively small number of projects, so that management may not be able to diversify sufficiently to offset large loss possibilities" [1969, p. 664].

Before the relative advantages of semi-variance in real estate investments are examined, it would be instructive to summarize the distinctions between the E-V and E-S_h models.[13] Comparing the utility functions underlying the E-V versus the E-S_h criterion, Mao demonstrated that whereas a quadratic utility function given by:

$$U(R) = a + bR + cR^2$$

justifies the E-V criterion, a utility function of the form represented by:

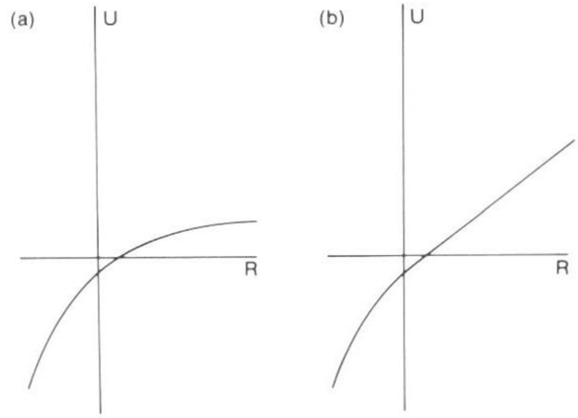
$$U(R) = a + bR + c\,[\mathrm{Min}(R-H), 0]^2$$

implies the E-S_h criterion of investment appraisal. Such a utility function is classified as a hybrid in that it is quadratic for $R \leq h$ and linear for $R > h$. Figure 1 depicts these utility functions: (a) represents the E-V utility function; and (b) represents the corresponding function for the E-S_h criterion.

The E-V or quadratic utility function describes an investor who avoids both extreme positive and negative returns. The hybrid utility function allows for aversion toward risk at low return and neutrality at high return. For a risk averse investor, the indifference curves that correspond to each of the utility functions would both be upward sloping and concave downward, but the shapes of the curves would differ. Coupled with the fact that the respective efficient sets depend in part on the measure of risk, different optimal portfolio selections may be expected under the E-V and E-S_h criteria of choice.

FIGURE 1

QUADRATIC UTILITY FUNCTION — HYBRID UTILITY FUNCTION



Other observations regarding the E-V versus E-S_h models pertain to cases where h takes on the values of E and zero. For investments with asymmetric distributions, the E-V and E-S_E criteria may indicate different optimal solutions. The E-V criterion is insensitive to the direction of skewness, whereas the E-S_E is prejudiced against distributions skewed to the left and insensitive to distributions skewed to the right. For investments involving only symmetric distributions, both models will produce the same optimal solutions which follows since $V = 2S_E$ for symmetric probability distributions.

The case in which the reference point h is zero is of particular interest. The risk measure S_0 focuses on the downside deviations from the zero profit point, that is, the distribution of losses. In evaluating investment alternatives, the E-S_0 criterion is prejudiced against investments having the greater scatter of points to the left of zero. Further, ranking investments by the E-V and E-S_0 criteria can produce different results regardless of whether the underlying distributions are symmetric or asymmetric.

The E-S_h model typically will have greater informational and computational requirements than the corresponding E-V model, which follows since computation of the portfolio semi-variance requires the joint probability distribution of investment returns, whereas portfolio variance could be computed from the variances of return of the individual underlying properties and the correlation coefficients of return between pairs of properties. The use of simulation may somewhat alleviate the problem.[14] Much additional research is required before the E-S_h model can be operationalized.

Skewness And Kurtosis

Financial writers have suggested that investors should evaluate investments on the basis of the third and fourth moments as well as the mean and variance of the distribution of returns.[15] The third moment, M_3, is given by $E[R-E(R)]^3$ and measures the skewness of a distribution; the fourth moment, M_4, given as $E[R-E(R)]^4$, measures the "tailedness" of a probability distribution and is often associated with kurtosis. Kurtosis is actually a measure of "peakedness" and refers to the normalized fourth moment rather than the raw fourth moment.

Although empirical evidence is mixed, usually it is believed that the investor's utility function should be an increasing function of M_3 — indicating a preference for positive skewness, and a decreasing function of M_4 — implying an aversion to "tailedness." Using data on individual securities and mutual funds Arditti [1967, 1971] found a statistically significant coefficient for skewness that indicated preference for positive skewness. In a more comprehensive study on mutual funds, Francis [1975] reported that previous evidence on the subject is sample dependent and inconclusive, and investors do not take cognizance of skewness. However, both the Arditti and Francis studies were limited since they tested pooled samples of multiple types of mutual funds rather than evaluating skewness preference behavior on subcategories of mutual funds classified by their stated goals.

Although there is the need for empirical and further conceptual investigations regarding skewness and kurtosis in real estate investment decisions, general conclusions regarding investor preferences even among mutual fund investors cannot be drawn. That such factors should be regarded in investment decisions is indeed justified by theory.[16] The nature of real estate investing — the more limited ability to diversify and the greater potential for large losses — would indicate a greater likelihood for asymmetric distributions of returns. The need to consider skewness may be greater for real estate than for securities.

Stochastic Dominance

Another alternative to mean-variance analysis is the stochastic dominance approach in which investment selection is conducted by employing efficiency criteria developed from the concepts of first, second and third degree stochastic dominance.[17]

First degree stochastic dominance (FSD) places no restrictions on investors' utility functions beyond the assumption that more wealth is preferred to less, that is, $(dU/dw) > 0$. To demonstrate, consider two probability distributions, $Q(X)$ and $R(X)$, where Q and R denote the cumulative distributions of two different investments or portfolios.[18] FSD states that investment Q will be preferred to R, independent of the concavity or convexity of the utility function if $Q(X) \leq R(X)$. This is equivalent to the condition that the two probability distributions do not intersect. Figure 2 demonstrates this graphically. The plots P, Q and R represent the cumulative probability functions for three distinct investments or portfolios. Investment P would be eliminated from the efficient set independent of the utility function. Investments Q and R would be retained, since they intersect and lie to the right of P.

Second degree stochastic dominance (SSD) assumes universal risk aversion or neutrality, $(d^2U/dW^2 \leq 0)$,

FIGURE 2

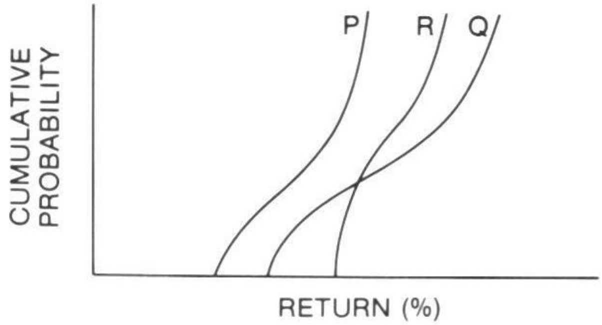
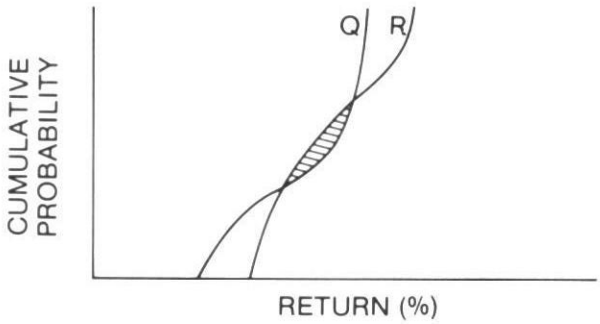


FIGURE 3



as well as rationality, $(dU/dW > 0)$. This criterion states that Q will be preferred over R if $\int_{-\infty}^{x} [R(t) - Q(t)]\,dt \geq 0$, for all x. Intersection between the cumulative probability distributions may occur, but the cumulative difference between R and Q must remain nonnegative over the entire domain x. SSD makes it possible to choose between investments that do not exhibit FSD. Figure 3 shows this graphically. The cumulative unshaded area for which $R(x) > Q(x)$ always exceeds the shaded area for which $R(x) < Q(x)$ over the entire domain of x.

When neither FSD nor SSD enable selection between two investments, the investor can use third-degree stochastic dominance (TSD). TSD also assumes that $(dU/dW > 0)$ and $(d^2U/dW^2 \leq 0)$. In addition, TSD requires that $(d^3U/dW^3 \geq 0)$. In essence, the prospect Q will be preferred to R if $\int_a^x \int_a^y [R(t) - Q(t)]\,d_t\,d_y \geq 0$, for all $x \in [a, b]$ and $\int_a^b [R(y) - Q(y)]\,d_y \geq 0$.[19]

Proponents of stochastic dominance argue for its theoretical superiority over the mean-variance method on the grounds that the dominance criteria place less restrictive constraints on the investor's utility function. Unlike the mean-variance criterion, stochastic dominance orderings are independent of the type of probability function under examination. However, in comparison to mean-variance, the application of stochastic dominance rules requires significantly more data. Dominance criteria require estimation of the entire probability function and larger numbers of comparisons are needed to derive the efficient set of portfolios. The recent development of efficient algorithms for applying stochastic dominance tests have partially alleviated these difficulties.[20]

The less restrictive utility aspects and the more comprehensive nature of stochastic dominance make it more appealing as an efficiency criteria for investments in general. Especially attractive for real estate is the attribute that stochastic dominance orderings do not depend on the type of probability function describing the investment or portfolio. For example, Feldstein [1969] and also Hanoch and Levy [1969] have shown that the E-V criterion requires that the two parameters of the distribution of returns be independent of one another, which limits the generality of the E-V criterion. Arguing that this restriction may not be so severe in practice,[21] Levy and Sarnat note that "... risk averse individuals tend to diversify their holdings, that is, they build portfolios of a number of securities. Moreover, mutual funds make relatively large portfolios of hundreds of individual securities readily available even to the small investor. To the degree that the returns of the individual securities are independent of one another, the return on relatively large portfolios should approximate a normal distribution." This conclusion is based on the Central Limit Theorem; and indirect empirical evidence shows that the distribution of returns to mutual funds investors does approximate the normal. The normal distribution depends on only two independent parameters — the mean and variance. Levy and Sarnat conclude that the E-V model provides an appropriate criterion for the mutual funds segment of the securities market. They state that "... to the extent that mutual funds provide a relevant proxy for investment portfolios in general, the statistical evidence suggests that the mean-variance criterion can provide an effective decision rule for most risk-averse investors..." [1972, p. 330].

These conclusions seem more applicable to securities investment than to real estate. While it can be assumed that real estate investors are risk averters and attempt to diversify, their abilities seem more limited. Although the real estate investment trusts (REITs) could provide the small investor with a means of diversification as mutual funds provide for security investors, it appears they do not. Investors may not consider investing in REITs as a substitute for direct real estate equity investment, that is, significant distinctions exist between investing in REITs and direct purchase of real estate. For example, REIT securities trade in relatively efficient and organized national security markets in comparison to the markets for direct real estate equity investment. In addition, direct real estate investment involves management costs normally not associated with REITs. The markets for REITs and direct real estate investment appear segmented and appeal to different classes of investors.

Therefore, it seems that neither mutual funds nor REITs provide a relevant proxy to direct real estate equity investment portfolios. The potential for asymmetric distributions of return and the absence of two independent parameter distributions in real estate investing seem greater.

Conclusions

Selected alternative risk measures and risk screening devices were evaluated with respect to their appropriateness and feasibility for assessing risk in real estate investments. Previous empirical studies in this area which have relied largely on variance type measures of dispersion have indicated that real estate returns in general are less risky than those for common stock. Deficiencies of the variance as a risk measure as well as significant distinctions between the return characteristics of real estate and securities justify a more extensive examination.

The alternative risk screening approaches investigated in this study included beta, semi-variance, skewness and kurtosis, and stochastic dominance. Results suggest that direct real estate equity investing for a given amount of wealth appears to offer lesser opportunity for diversification and greater potential for large losses than for securities investment. A greater likelihood for asymmetric distributions of return for real estate as compared with securities is suggested. To the extent that asymmetry occurs in real estate returns, the semi-variance, skewness and kurtosis, and stochastic dominance approaches appear more appealing in concept than the variance. The measure, S_0, the semi-variance with zero profit as the point of reference, is appealing for real estate even when the distributions of return are symmetric. The more sophisticated approaches entail greater data and computational problems in application than the mean-variance technique. Since real estate portfolios tend to involve fewer distinct assets and require consideration of fewer pre-selection alternatives than security portfolio selection, these problems are not necessarily prohibitive.

The tractability of the systematic risk or beta concept has definite advantages for real estate investing although the nature of current real estate assets and markets severely limits the applicability of the CAPM in direct equity real estate investment.

Conclusions were based on a conceptual analysis of the characteristics of real estate assets. These characteristics are the lumpiness (largeness), indivisibility, illiquidity, extensive leveraging and greater management role inherent in real estate investment. Extensive empirical research is still needed.

NOTES

1. For a comprehensive summary of the deficiencies of the variance as a risk measure, see Levy and Sarnat [1970, 1155].

2. Empirical evidence of skewness in security portfolios was reported by Arditti [1971].

3. For a conceptual discussion, see Pellatt [1972].

4. Thirty-three properties from Pacific Mutual Life Insurance Company's real estate portfolio were sampled; the balance was properties held by savings and loan associations.

5. An assumed constant compound rate of growth tends to produce a fairly constant annual holding period return and potentially biases downward the estimated total variance of return for the real estate assets.

6. Risk and uncertainty, while not strictly equivalent [Knight, 1921, Ch. VII], generally are interchanged in investment literature. This follows from the assumption that it is always possible to convert uncertainties into risks by introducing subjective probabilities. For a more elaborate discussion, see Levy and Sarnat [1972, 189-191].

7. See Sharpe [1970].

8. To more adequately demonstrate the distinction between systematic and unsystematic or random risk, consider the variance of equation given by $\sigma^2_{R_i} = \beta_i \sigma^2_{R_m} + \sigma^2_{\eta_i}$. The total variation in returns to the ith asset, $\sigma^2_{R_i}$, is represented as the sum of the systematic variation, $\beta_i \sigma^2_{R_m}$, and the random or unsystematic variation, $\sigma^2_{\eta_i}$. The unsystematic risk, $\sigma^2_{\eta_i}$, entails the potential for return variations due to labor strikes, fires and other occurrences, and is considered diversifiable. Systematic risk, $\beta_i \sigma^2_{R_m}$, reflects potential variations in return caused by events affecting the general market for all securities. Examples would include changes in interest rates or government taxing policies. Within a given economy the systematic variation is regarded as nondiversifiable. Mathematically this is shown as follows: let $R_p = \sum_{i=1}^{n} X_i R_i$ represent the return to the portfolio where X_i (for all i=1, ... n) denotes the proportion of total wealth allocated to the ith asset. The variance of portfolio return could then be shown to be

$$\sigma^2_{R_p} = (\sum X_i \beta_i)^2 \sigma^2_{R_m} + \sum X_i^2 \sigma^2_{\eta_i}.$$

Assuming for concreteness that $X_i = 1/n$, let $\beta = \sum \frac{\beta_i}{n}$ represent the average β for the portfolio and $\sigma^2_{\eta} = \sum \sigma^2_{\eta_i}/n$, the average portfolio random risk. Thus, $\sigma^2_{R_p} = \beta^2 \sigma^2_{R_m} + \sigma^2_{\eta}/n$. As $n \rightarrow \infty$ (that is, diversification is increased), $\sigma^2_{R_p} = \beta^2 \sigma^2_{R_m}$, or random risk is eliminated.

9. See the comments by Williams and Findlay [1974, 359] regarding the dissertation of Friedman [1972].

10. Individual real estate projects such as a condominium complex, a shopping center or an office building may often cost millions of dollars.

11. This assumes that risk reduction through diversification occurs through an increasing number of distinct assets held in the portfolio. It may be possible that risk reduction occurs more quickly (that is, with fewer distinct assets) in real estate than with common stocks. This is not apparent and cannot be assumed.

12. See "Money Management," *Business Week* (October 11, 1976), 100-109.

13. For a more detailed discussion, refer to Mao [1970].

14. Simulation for the purpose of generating probability distributions of return is receiving increased attention in real estate. See Pyhrr [1973] and Findlay, Tarantello and Messner [1976].

15. See Arditti [1967, 1971] regarding the third moment, and Tuncer [1975] regarding the fourth moment.

16. See Arditti [1967].

17. See Levy and Sarnat [1970, 1972] and Whitmore [1970].

18. This discussion draws largely from Levy and Sarnat [1970].

19. Refer to Whitmore [1970, 457-458].

20. See Porter, Wart and Ferguson [1973].

21. See Levy and Sarnat [1972, 325-330].

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