APPENDIX A

Demonstration of the Proposed Methodology—A Hypothetical Example

We concluded that the specification of two parameters, the correlation and beta coefficient, was an appropriate description of an operational investment policy which uses the latest concepts in portfolio theory and provides clear advantages over heretofore traditional investment practices. Once the technique is understood, the application of the methodology becomes a routine matter of calculation.

In order to demonstrate the steps involved we will consider the comparison of a hypothetical real estate investment portfolio with an index of New York Stock Exchange common stocks. It may be helpful to think of the situation where a pension fund manager is considering the inclusion of our hypothetical real estate portfolio in a large pool of invested funds. The pension fund manager might like answers to questions such as:

> Will the proposed portfolio exceed the expected rate of return on the current pension fund assets?
> Does the inclusion of this portfolio contribute to a reduction in the overall riskiness of the current investment portfolio?
> What is the extent to which this proposed portfolio is diversified?
> Is the risk premium sufficiently large for a given level of risk for inclusion within our total portfolio?
> Is the manager of the proposed portfolio demonstrating real skill in picking a superior portfolio or is he just lucky?
> Does the proposed portfolio fall within the constraints imposed upon me, as the pension fund manager, with regard to the allowable beta and correlation coefficients to warrant further investigation of this investment opportunity?
> Has our portfolio (the hypothetical portfolio) deviated so far from past behavior to require some restructuring to bring it back into line with our explicit investment objectives?

The seven steps to arrive at the beta and correlation coefficients that can shed light on these questions follow.

Step 1: Determine the quarterly wealth relatives of the portfolio under investigation, X_{iq}, and the comprehensive index, X_{jq}.

Let us analyze the performances of a hypothetical real estate portfolio and NYSE common stocks over an identical three-year time period. *Tables A* and *B* illustrate computations required for later use beginning from a presumed sequence of wealth relatives for the real estate investment and from the reported monthly rates of return for the NYSE stocks. The real estate wealth relatives include all cash distributions, all expenditures to cover operating losses and periodic revaluations of the market value of the portfolio. The monthly returns on stocks reported by the Center for Research in Security Prices were first converted to quarterly returns from which the wealth relatives were deduced.

For convenience of notation we have denoted variables associated with the hypothetical real estate investment by the subscript i and variables of the market by the subscript j.

TABLE A

HYPOTHETICAL REAL ESTATE INVESTMENT

Quarter		Column I (X_{iq}) Wealth Relative	Column II Quarterly Rate of Return	Column III ($\ln X_{iq} - \ln \overline{X}_i$)	Column IV ($\ln X_{iq} - \ln \overline{X}_i)^2$
1970	1st	1.01131	0.01131	-0.03098	0.00096
	2nd	1.00123	0.00123	-0.04100	0.00168
	3rd	1.10836	0.10836	0.06066	0.00368
	4th	0.98001	-0.01999	-0.06242	0.00390
1971	1st	1.01652	0.01652	-0.02584	0.00067
	2nd	1.07617	0.07617	0.03118	0.00097
	3rd	0.99990	-0.00010	-0.04233	0.00179
	4th	1.07358	0.07358	0.02877	0.00083
1972	1st	1.13294	0.13294	0.08259	0.00682
	2nd	0.99358	-0.00642	-0.04867	0.00237
	3rd	1.09637	0.09637	0.04978	0.00248
	4th	1.04133	0.04133	-0.00173	0.00000
				$\Sigma =$	0.02615

TABLE B

NEW YORK STOCK EXCHANGE STOCKS

Quarter		Column I (X_{jq}) Wealth Relative	Column II Quarterly Rate of Return	Column III ($\ln X_{jq} - \ln \overline{X}_j$)	Column IV ($\ln X_{jq} - \ln \overline{X}_j)^2$
1970	1st	0.96654	-0.03346	-0.05488	0.00301
	2nd	0.79864	-0.20136	-0.24570	0.06037
	3rd	1.16928	0.16928	0.13554	0.01837
	4th	1.09049	0.09049	0.06578	0.00433
1971	1st	1.10125	0.10125	0.07559	0.00571
	2nd	1.00148	0.00148	-0.01937	0.00038
	3rd	0.98843	-0.01157	-0.03249	0.00106
	4th	1.03662	0.03662	0.01511	0.00023
1972	1st	1.06567	0.06567	0.04275	0.00183
	2nd	0.99741	-0.00259	-0.02344	0.00055
	3rd	1.02212	0.02212	0.00103	0.00000
	4th	1.06284	0.06284	0.04009	0.00161
				$\Sigma =$	0.09745

Step 2: Compute the respective mean quarterly wealth relatives, X_i and $\bar{X}_j$.

The mean quarterly wealth relatives are represented by the geometric mean of the individual quarterly wealth relatives plus 1.00. The results are as follows:

for the hypothetical real estate portfolio

$$\bar{X}_i = \left[\prod_{q-12}^{q} X_{iq} \right]^{\frac{1}{12}} = 1.04313$$

and for the market index

$$\bar{X}_j = \left[\prod_{q-12}^{q} X_{jq} \right]^{\frac{1}{12}} = 1.02107$$

These results are then utilized in Columns III and IV of *Tables A* and *B* to arrive at the numbers shown therein.

Step 3. Compute the respective mean quarterly rates of return, R_i and R_j.

From the mean quarterly wealth relatives it follows directly that the mean quarterly rates of return for the period under investigation are:

$\bar{R}_i = \bar{X}_i - 1 = 0.04313$ for the real estate portfolio

and

$\bar{R}_j = \bar{X}_j - 1 = 0.02107$ for the market index

Upon inspection of *Table A* we can see that individual quarterly rates of return for the real estate portfolio ranged from a high of 13.294% to a low of -1.999% about a mean of 4.313%. Corresponding results for the stock market index were individual quarterly rates of return ranging from a high of 16.928% to a low of -20.136% about a mean of 2.107%. The market index thus experienced both a lower mean return than our hypothetical portfolio and a much wider range in individual quarterly results.

Step 4: Compute the respective variance and standard deviation of quarterly returns, Var_i and σ_i and Var_j and σ_j.

The calculations of variance and standard deviation of returns about the mean are shown, in part, in Column IV in *Tables A* and *B*. The final computations are as follows:

for the real estate portfolio

$$Var_i = \frac{\Sigma\,(\ln X_{iq} - \ln \bar{X}_i)^2}{n} = \frac{0.02615}{12} = 0.00218$$

$$\sigma_i = \sqrt{Var_i} = 0.04668$$

and for the market portfolio or index

$$\text{Var}_j = \frac{\Sigma\ (\ln X_{jq} - \ln \overline{X}_j)^2}{n} = \frac{0.09745}{12} = 0.00812$$

$$\sigma_j = \sqrt{\text{Var}_j} = 0.09012$$

Step 5: Compute the covariance of real estate portfolio returns with market returns, Cov_{ij}.

Table C shows the computations required to arrive at the covariance. Columns I and II of *Table C* are taken directly from Column III of both *Tables A* and *B*.

TABLE C

Quarter		Column I ($\ln X_{iq} - \ln \overline{X}_i$)	Column II ($\ln X_{jq} - \ln \overline{X}_j$)	Column III Col. I x Col. II
1970	1st	-0.03098	-0.05488	0.00170
	2nd	-0.04100	-0.24570	0.01007
	3rd	0.06066	0.13544	0.00822
	4th	-0.06242	0.06578	-0.00411
1971	1st	-0.02584	0.07559	-0.00195
	2nd	0.03118	-0.01937	-0.00060
	3rd	-0.04233	-0.03249	0.00138
	4th	0.02877	0.01511	0.00043
1972	1st	0.08259	0.04275	0.00353
	2nd	-0.04867	-0.02344	0.00114
	3rd	0.04978	0.00103	0.00005
	4th	-0.00173	0.04009	-0.00007
				Σ = 0.01979

The final computation of the covariances relies upon the product of the numbers shown in Columns I and II which result in Column III of *Table C*. Thus, the covariance is:

$$\text{Cov}_{ij} = \frac{\Sigma\ (\ln X_{iq} - \ln \overline{X}_i)\ (\ln X_{jq} - \ln \overline{X}_j)}{n} = \frac{0.01979}{12} = 0.00165$$

Step 6: Determine the measure of sensitivity of rates of return on the hypothetical real estate portfolio to the rates of return on the market, β_i.

The beta coefficient derives directly from results of Steps 4 and 5 shown above. Thus, the beta coefficient for this real estate portfolio would be:

$$\beta_i = \frac{\text{Cov}_{ij}}{\text{Var}_j} = \frac{0.00165}{0.00812} = 0.20320$$

Step 7: Determine the correlation coefficient and the coefficient of determination for this portfolio, ρ_{ij} and R^2.

When inserting the results of our hypothetical portfolio into the expanded and rearranged equation for the covariance we are able to determine its correlation with the market over the three year period under investigation.

$$\rho_{ij} = \frac{Cov_{ij}}{\sigma_i \sigma_j} = \frac{0.00165}{(0.04668)(0.09012)}$$

$$\rho_{ij} = \frac{0.00165}{0.00421}$$

$$\rho_{ij} = 0.39192$$

Since the coefficient of determination is merely the square of the correlation coefficient we have the simple calculation:

$$R^2 = \rho^2_{ij} = (0.39192)^2$$

$$R^2 = 0.15360 \text{ or } 15.360\%$$